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13. ABSTRACT (Maximum 200 words)

The main goal of the research is to understand nonlinear processes in natural phenomena. There is a strong emphasis on nonlinear optics, a subject which is relatively young and extremely rich in scientific and technological potential. Turbulance in optics, the study of the complex space-time patterns and defects which appear in feedback cavities and counterpropagating beams, is more analytically tractable than its counterpart in fluids, and is currently attracting international attention. It is the subject of our latest workshop.

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1986-1989

ARIZONA CENTER FOR MATHEMATICAL SCIENCES

FINAL REPORT

Contract No. F49620-86-C-0130

October 1986 — September 1989

QUARTERLY REPORT

Contract No. FQ8671-900589

December 1989 — March 1990

Department of Mathematics

University of Arizona

Tucson, Arizona 85721

March 1990

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TABLE OF CONTENTS

A. SUMMARY

The Arizona Center for Mathematical Sciences (ACMS) was begun in 1986 with support from the Air Force Office of Scientific Research under the University Research Initiative Program. It is housed on the eighth floor of the Gould-Simpson Building adjacent to the Mathematics Building and enjoys close ties with the Departments of Mathematics, Physics, Aerospace and Mechanical Engineering, Optical Sciences and the Applied Mathematics Program at the University of Arizona.

The main goal of the research is to understand nonlinear processes in natural phenomena. There is a strong emphasis on nonlinear optics, a subject which is relatively young and extremely rich in scientific and technological potential. Turbulence in optics, the study of the complex space-time patterns and defects which appear in feedback cavities and counterpropagating beams, is more analytically tractable than its counterpart in fluids, and is currently attracting international attention. It is the subject of our latest workshop.

There is little doubt that the Center has been enormously successful. The first rate regular visitors (Zakharov, Pomeau, Coullet, Rand, Firth, Moloney), the superhigh quality of recent postdoctoral fellows (Passot, Liverani, Lega), the quality of the graduate students, the reputation of the workshops, the interactions with AFWL at Kirtland and CNLS at Los Alamos, the productivity of the faculty (over 200 publications since 1986), all attest to this success. We are extremely grateful to the Air Force Office of Scientific Research who support the activities of the Center and hope they feel that their confidence in us has been amply justified.

B. MISSION

The primary goal of the Center is to provide an environment for research and learning in the Mathematical Sciences. Its basic research themes are the modelling, understanding and applicability of nonlinear processes in optics, fluids, neural networks, and random distributed systems with continuing investigations into pattern dynamics, percolation, behavior of lattice gases, nonlinear stability, low dimensional chaos, turbulence, dynamical systems and the nature of integrable systems of differential equations.

The research takes place at three levels. First, there is the long-range, ongoing research focusing on the themes listed above by permanent members of the faculty, colleagues with regular visiting arrangements, postdoctoral fellows and graduate students.[1] Second, there are collaborations

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[1]In contrast to the NSF Centers at Berkeley, Minnesota and Santa Barbara, this is our principal activity.

with one-time visitors from other universities, national laboratories and Air Force research centers who come for extended visits in connection with our special years' programs. Third, there is a regular series of workshops (fifteen in the last three years) which (i) address areas which look to the future, and which, in our judgment, are rich in promise and potential; (ii) have a pedagogic nature and are designed to communicate the importance and promise of several of the fields mentioned above to undergraduate and potential graduate students and to other scientific colleagues; and (iii) are special purpose and have a relatively narrow focus on topics of particular interest to Center faculty and collaborators.

The learning takes place at all levels. The breadth of activity and spectrum of interest and talent among visiting colleagues serves to stimulate interdisciplinary work and promote the cross fertilization of ideas. Graduate students interested in applied mathematics enjoy a unique environment in which they can experience first hand the unity in the approaches (modelling, simulation, analysis, and involvement in experiments) with which mathematical scientists tackle a diverse set of problems from all areas of the physical sciences. There are several ongoing weekly working seminars in addition to regular departmental colloquia. These are in the areas of applied analysis, computation, dynamical systems, nonlinear optics, neural networks, integrable systems, and mathematical physics. Finally, we mention that we have had great success with our introductory workshops, the purpose of which is to expose undergraduate seniors and potential graduate students to the exciting challenges of applied mathematics.

In short, the support of the Air Force Office of Scientific Research under the University Research Initiative Program has provided the Center with the flexibility, resources and the critical mass in the sub areas of concentration to satisfy an important criterion for a large scale research effort, namely that the whole is greater than the sum of its parts.

C. REASONS FOR SUCCESS

There are several factors, beyond the financial support, which have contributed to the Center's success. The first is that there was already a strong core of high quality, highly active faculty members whose research interests became the main themes of research in the Center. Second, there is a strong university and departmental commitment to the applied areas. This support complements the support provided by the Air Force. The Applied Mathematics Program at Arizona has existed for nearly fifteen years, enjoys an international reputation, is a source of excellent students, and provides the framework for genuinely interdisciplinary interactions. The University made a commitment of twelve new positions to the Department of Mathematics, including post-doctoral fellows, doubled the number of teaching assistantships, provided funds for special years a year before the Center was inaugurated. The University also has provided space for the Center in a beautiful setting on the northern-facing side

of the eighth floor of the Gould-Simpson Building. In fact, there are current plans to build a new Mathematics-Physics building in which one whole floor will be devoted to Center-like activities. In addition, the Mathematics Department takes a particular pride in and has a continuing commitment to its applied components. This makes Arizona somewhat unique. Third, with a combination of University and Air Force support, the Center was successful in building a first rate computational environment, with easy access of permanent and visiting faculty to SUN and IRIS workstations, mini-supercomputers (Convex 240), the CRAY-II at Kirtland Air Force Base and the CRAY-YMP at Pittsburgh. A key factor in the computer environment is the presence of outstanding support personnel. We have been most fortunate indeed in securing the services of Robert Indik, a Princeton Ph.D. in Number Theory, who is not only a first rate software consultant but is also actively involved in many Center projects, particularly in Optics. We also have an outstanding Computer Manager supported by the Mathematics Department, Bob Condon, who was principally responsible for setting up our network.

D. CENTER FACULTY AND STAFF

Partial biographies of the various members are given in Section VI.

Director   Alan C. Newell, Professor and Chairman, Dept. of Mathematics

Center Faculty   Bruce Bayly, Assistant Professor, Mathematics
                          Moysey Brio, Assistant Professor, Mathematics
                          Kwok Chow, Assistant Professor, Mathematics
                          Nicholas Ercolani, Associate Professor, Mathematics
                          Hermann Flaschka, Professor, Mathematics
                          Brenton LeMesurier, Assistant Professor, Mathematics
                          David Levermore, Associate Professor, Mathematics
                          David McLaughlin, (currently at Princeton University)
                          Charles Newman, Professor, Mathematics (presently visiting Courant Institute)
                          Maciej Wojtkowski, Associate Professor, Mathematics
                          Lai-Sang Young, Associate Professor, Mathematics

External Faculty Who Spend Regular, Extended Periods at the Center

Pierre Coullet, University of Nice, France
William Firth, University of Strathclyde, Glasgow
Jerry Moloney, Heriot-Watt University, Edinburgh
Yves Pomeau, Ecole' Normale Superieure, Paris
David Rand, Warwick University, England
Sacha Rubenchik, Institute of Automation and Electrometry, Novosibirsk, USSR
Volodja Zakharov, Institute of Theoretical Physics, Moscow

Post-Doctoral Fellows and Visiting Faculty Closely Associated with the Center

Douglas Abraham, (Ph.D., 1968, King's College, Statistical Mechanics), 1987-1988, currently at Oxford University

Alejandro Aceves, (Ph.D., 1988, University of Arizona, Nonlinear Optics), 1988-1989, currently at University of New Mexico

Wayne Arter, (Ph.D., 1983, Trinity College, Cambridge, Computational Science), 1986-1987, currently at Culham Lab (England)

David Barsky, (Ph.D., 1987, Rutgers University, Statistical Mechanics), 1988-1989, currently at University of California at Davis

Andrew Bernoff, (Ph.D., 1985, Trinity College, Cambridge, Nonlinear Dynamics), 1988-1989, currently at University of California at Berkeley

Jean-Guy Caputo, (Ph.D., 1986, University of Grenoble, Dynamical Systems), 1987-1988, currently at Ins a de Rouen (France)

Martin Casdagli, (Ph.D., 1985, Warwick University, England, Dynamics), 1986-1987, currently at Queen Mary College, London

Patrick Dunne, (Ph.D., 1987, M.I.T., Hydrodynamic Stability, Nonlinear Waves), 1987-1988, currently at Ballinahown (Dublin)

Alecsander Dyachenko, (Ph.D., 1988, Moscow Phy Tech Inst, Physics), 1990-, from USSR Scientific Council

G. R. Grimmett, (Ph.D., 1974, Mathematical Institute, Oxford, Probability), 1987-1988, currently at University of Bristol (England)

Wanda Henry, (Ph.D., 1988, Australian National University - I.A.S., Optics), 1988-1989, currently at King's College (Cambridge)

Joceline Lega (Ph.D., 1989, Universite de Nice, Physics), 1990-, from Lab de Phy Theor (France)

Liverani, Carlangelo (Ph.D., 1988, Rutgers University, Mathematics), 1990-, from Rutgers University

Alistair Mees, (Ph.D., 1973, Cambridge University, Dynamical Systems), 1987-1988, currently at University of Western Australia

Edward Overman, II, (Ph.D., 1978, Ohio State University, Computational Science), 1986-1987, currently at Ohio State University

Thierry Passot, (Ph.D., 1987, Observatoire de Nice, Turbulence, Painlevé Analysis), 1987-, from CNRS (France)

Andrei Pushkarev, (Ph.D., 1988, Moscow Phy Tech Inst, Physics), 1990-, from USSR Scientific Council on Complex Problems

Ron Sawatzky, (Ph.D., 1987, University of Alberta, Wave Propagation), 1987-1988, currently at University of Alberta

Vadim Shvets, (Ph.D., 1980, Inst for Theor Phy, USSR, Physics), 1990-, from USSR Scientific Council on Complex Problems

M'Hamed Souli, (Ph.D., 1984, Universite de Nice, Computational Science), 1988-1989, currently at University of Grenoble

Wabnitz, Stefan, (Ph.D., Applied Physics,), 1990-, from Fondazione Ugo Bordoni (Italy)

Winful, H. G., 1990-, from University of Michigan

Henryk Zoladek, (Ph.D., 1983, Moscow State University, Bifurcation Theory), 1987-1988, currently at Warsaw University

Center Staff   Administrative Assistant: Mary Bollschweiler
                      Computer Software Specialist: Robert Indik
                      Computer Manager: Robert Condon

E. AREAS AND PROJECTS

Here we list the main areas of activity. Detailed descriptions of the projects are given in Part II, Areas of Current Research. We emphasize that there is much interplay and overlap between the various projects.

Nonlinear Optics

- Turbulence, Defects, Spatial Patterns in Optics

- Counter-Propagating Beam Instabilities in Bulk Media

- Counter-Propagating Beam Instabilities in Optical Fibers

- Dynamics of Free-Running and Injection-Locked Laser Diode Arrays

- Diffraction/Diffusion Mediated Instabilities in Self-focusing/Defocusing
  Nonlinear Amplifying Media

- Coupled-Wave Interactions in Extended Media

- Theoretical Study of Optical Phase Conjugation in Stimulated Brillouin Scattering

- Nonlinear Optical Switching at Multiple Interfaces

Turbulence and the Nature of Spatial-Temporal Complexity in PDE'S

- The Role of Collapse Structures in Nonlinear Physics

- Coherence and Chaos in Near-Integrable PDE's

- Local Inertial Manifolds

- Exactly Soluble Models for the Propagation of Oscillations

- Local Inertial Manifolds and Adaptive Basis Schemes for Dynamical Systems

- Turbulence, Hydrodynamic Stability Theory and Dynamics

- Turbulence in Compressible Flows and the Incompressible Limit of the
  Navier-Stokes Equations

- The Role of Defects in creating Strong Turbulence

- Three-Dimensional Euler Equations

- Analytical and Topological Studies of Singularity Formation in Euler Equations

- Shear and Turbulent Convection

Fluids, Fronts, Stability and Transition

- Convecting Patterns

- Nonlinear Dynamics

- Large Scale Instabilities in Tri-dimensional Compressible Flows

- Subgrid Scale Modeling in Two-Dimensional MHD Turbulence

- Caustics in Convection Patterns

- Evolution of Localized States and Fronts in Non-Gradient Flow Systems

- Front Propagation

- The Theory of Compressible Fluids

- Kinetic Theory of Fluids

- Laminar-Turbulent Transition

Computational Science

- Phased Diode Laser Arrays

- Coupled-Wave Interactions in Extended Nonlinear Optical Media

- Discontinuous Solutions of Hyperbolic Systems

- Lattice Gas Hydrodynamics

- Self Focusing Phenomena in Lasers

Integrable Systems and Geometry

- Gauge Field Constructions

- Construction of Constant Mean Curvature Surfaces

- Topological Classification of Integrable PDE

- Painleve Analysis of the Toda Lattice

- Momentum Mappings

- Nonlinear Poisson Structures

- Topology of Level Surfaces

Random Distributed Systems

- Exactly Soluble 3D Random Surface Model

- Ising Spin Systems

Dynamical Systems

- Hamiltonian Dynamical Systems

- Random Perturbations of Dynamical Systems

- Statistical Properties of Orbits Near Strange Attractors

F. WORKSHOPS

The Workshops have each focused attention on new challenges and stressed the connections which exist between various mathematical sciences, connections which often are ignored but which provide dividends when pursued. Examples are the fundamental role of nonlinearity in optics, the interplay between the coherence of solitons and the scattering (Anderson localization) effects of randomness, and the value in looking at numerical algorithms from the perspective of dynamical systems.

~ *Numerical Solutions of Nonlinear Differential Equations, January 1987*

~ *Random Schrödinger Equations, February 1987*

~ *State of the Art Developments in Nonlinear Optics, March 1987*

~ *Singularities in Nonlinear Partial Differential Equations, March 1988*

~ *The Lagrangian Picture of Fluid Dynamics, October 1988*

~ *Space-Time Complexity in Nonlinear Optics, March 1990*

In addition, several special purpose workshops were held with colleagues from the Air Force Weapons Laboratory at Kirtland Air Force Base and the Center for Nonlinear Studies at Los Alamos.

~ *Kirtland Air Force Base Workshop, December 1986, at Arizona*

~ *Kirtland Air Force Base Workshop, March 1988, at Kirtland*

~ *Kirtland Air Force Base Workshop, October 1988, at Arizona*

~ *Kirtland Air Force Base Workshop, October 1989, at Kirtland*

~ *Los Alamos Days Conference, October 1986, at Arizona*

~ *Los Alamos Days Conference, January 1988, at Los Alamos*

~ *Los Alamos Days Conference, February 1989, at Arizona*

~ *Los Alamos Days Conference, December 1989, at Los Alamos*

We have had four workshops for undergraduates.

- *I Annual Undergraduate Workshop in Nonlinear Science, March 1987*

- *II Annual Undergraduate Workshop in Nonlinear Science, March 1988*

- *III Annual Undergraduate Workshop in Nonlinear Science, March 1989*

- *IV Annual Undergraduate Workshop in Nonlinear Science, March 1990*

G. COLLABORATIONS WITH COLLEAGUES AT AFWL AT KIRTLAND

While there is mutual interest in several topics, the topic on which there is a specific focus at present is the problem of stimulated Brillouin back scattering. This effort involves software development for the study of stimulated Brillouin scattering (SBS) in nonlinear optical media. These numerical schemes for solving the appropriate nonlinear partial differential equations will be extended to include wide-angle beams, transient pulse effects with transverse self-focusing and pump depletion. The software under development addresses the specific research interests of the laser theory group at AFWL; the objective of the research is to provide a portable and flexible computer code for their use. A joint theoretical study of phase conjugation in SBS systems is also being carried out.

H. SPECIAL YEAR PROGRAMS

As part of its commitment to maintaining its leadership in algebra, nonlinear analysis, and applied probability and to developing new strengths in computational science and geometry, the Department has held a series of special year programs which are designed to bring to Arizona both senior and junior visitors for extended visits.

In Spring 1986, there were two programs, one on Algebraic Geometry with an emphasis on Abelian varieties and another on Chaos and Turbulence. In 1986-87, our focus was on Computational Mathematics. In Spring 1987, in collaboration with the Center for Complexity, the emphasis was on Probability and Applications and brought together researchers interested in Statistical Mechanics, Image Processing, Random Media, Chaotic Dynamics and Probabilistic Number Theory. In 1989-90, a special year in Biomathematics was held. Currently in Spring 1990, we have a special focus on the role of collapse structures in optics and plasmas.

I. THE TRAINING OF STUDENTS AND POSTDOCTORAL FELLOWS

A very important part of the Center mission is the training of graduate students and postdoctoral fellows. Since the beginning of the Center, it has supported 22 students and 19

postdoctoral associates. There are several key ways in which the Center has greatly improved the learning enviornment. First, there is a critical mass of people (faculty, postdocs and students) in each of the areas of emphasis who meet on a regular basis in working seminars. Second, the constant stream of first rate visiting colleagues serves as a continuing stimulus and exposes our students to a broad variety of challenges in the Mathematical Sciences. Moreover, students directly experience the parallel transport of ideas from one area to another. This involvement in projects and discussion acts as a catalyst to bring out the best in the student and to give him the widest possible exposure to all areas of the Mathematical Sciences. Third, because we have been able to develop a first rate computer environment (with advanced laboratory courses in computational science) and because our students are exposed to experimental work by our colleagues in Optical Sciences and Aerospace and Mechanical Engineering, our young people are taught the value of the interplay between the three modes of modern investigation, experiment, analysis and numerical simulation.

J. THE FUTURE

The center has been funded for another three-year period beginning December 1989. We have established the research, computational and training environment to the point where the ACMS at Arizona has become a very attractive place to work and learn. This reputation has led to the planned extended visits of several outstanding mathematical scientists. Yves Pomeau will be a regular member of the Center, spending at least three months a year here. Volodja Zakharov and Sacha Rubenchik plan to bring several of their young colleagues to Arizona for the Spring 1990. We expect them to visit again next year. William Firth, an optical scientist, will visit us on a regular basis. David Rand and Jerry Moloney will continue their regular visits.

We also see the strengthening of the link with AFWL at Kirtland Air Force Base. There is considerable overlap of research interests in nonlinear optics and tremendous benefits for both sides in continuing the collaboration. We believe we have made substantial progress in cracking some of the problems of mutual interest.

In connection with the importance of computation, the Department has made the area of Computational Science a top priority. This means that in addition to the many young people we have here, we will be recruiting a first rate, senior scientist in this area.

With the continued support from the Air Force, the University and the Department, we believe that ACMS has rapidly become one of the National Centers in the Mathematical Sciences.

Filament Patterns in a Feedback Ring Cavity
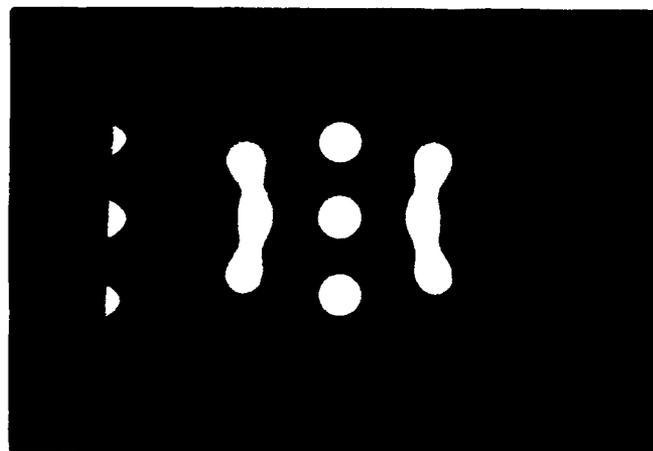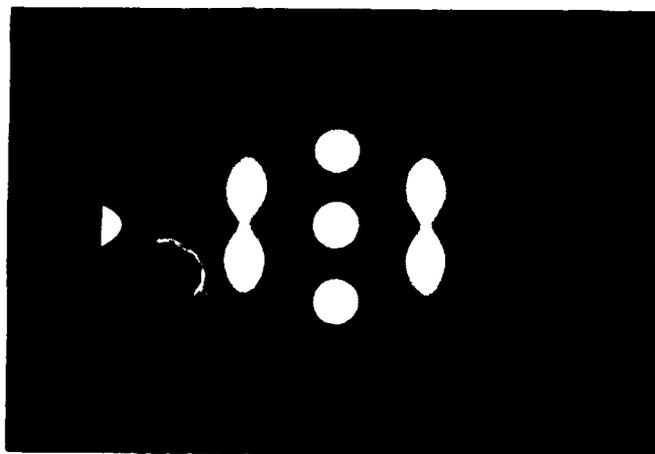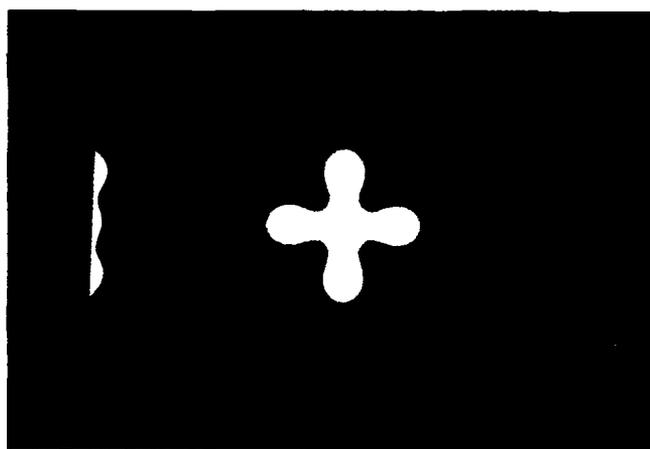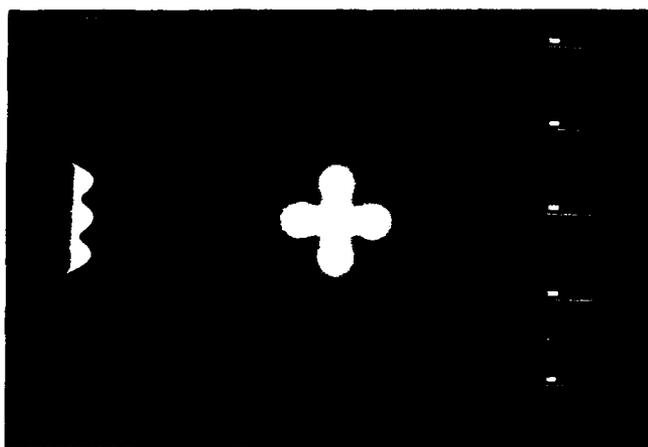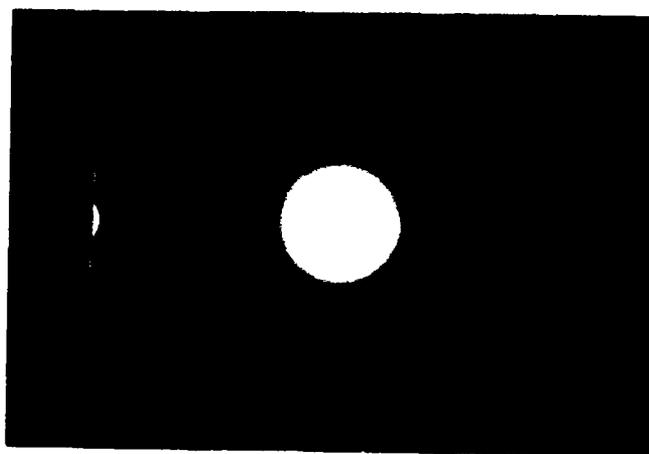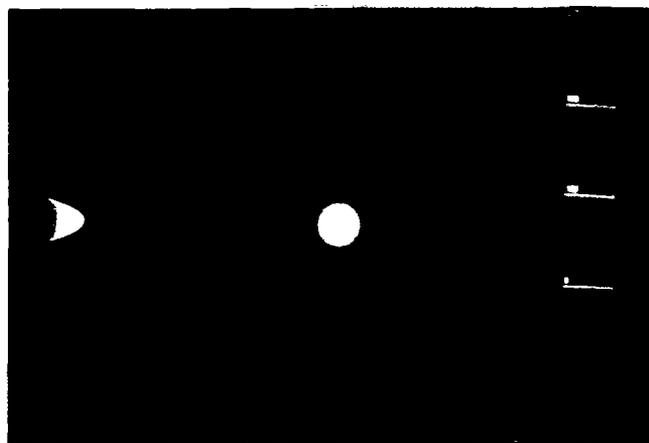
II. AREAS OF CURRENT RESEARCH

A. NONLINEAR OPTICS
(Moloney, Newell)

Turbulence, Defects and Spatial Patterns in Optics

The Center's activity in nonlinear optics has broadened significantly in scope over the past years. A key to this rapid growth has been our success in attracting leading researchers in optics worldwide as active participants in the Center. The mix of senior and junior faculty, postdoctoral fellows and graduate students coupled with the additional optics expertise input from our visitors, colleagues at the Optical Sciences Center and at Kirtland AFB provides the critical base essential to carrying through ambitious wide-ranging optics research activity. Among the long-term visitors who will play a key role in future research activities are: W. J. Firth of Strathclyde University (counter-propagating beam induced instabilities), S. Wabnitz of the Fondazione Ugo Berdoni, Rome (nonlinear optics), H. G. Winful of Michigan (laser diode arrays) and P. Coullet of Nice (patterns and optical vortices).

The workshop on "Space-Time Complexity in Nonlinear Optics" affords these and other leading figures in nonlinear optics and mathematics an opportunity for discussion and debate on current outstanding problems and theoretical challenges at the forefront of modern optics. This workshop is a follow-up to our highly successful one on "State of the Art Developments in Nonlinear Optics," held in 1987.

In addition to ongoing research projects, the Center's activity is migrating towards optics problems wherein finite material response times play a critical role in inducing a variety of sideband instabilities. Areas in which we anticipate that rapid progress will be made are (1) stability of phased diode laser arrays under free-running and injection-locking conditions, (2) self-defocusing induced instabilities leading to complex spatial pattern formation in nonlinear amplifying media, (3) counter propagating beam induced instabilities in passive and amplifying nonlinear optical media, and (4) bright/dark solitary wave patterns induced on laser beams in optical feedback structures. These areas offer a rich class of mathematically and computationally challenging research problems which will have a significant impact on future developments within nonlinear optics as a whole.

A.1. Counter-Propagating Beam Instabilities in Bulk Media
(Firth, Indik, Moloney, Newell, Wright)

Sideband-induced instabilities associated with indirect nonlinear spatial gratings have been studied at a preliminary level for counter-propagating infinite plane waves with finite material response at one extreme and for counter-propagating Gaussian (one transverse dimensional) beams in instantaneously

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†References in text fall into two categories. One is the set of publications of the Center (Section VII). The other category includes references to outside work relevant to the research. The latter are identified with an asterisk.

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responding nonlinear media (Firth) [1,2,3]. Both Indik and Wright (Optical Sciences Center) have been working in collaboration with W. J. Firth (Strathclyde) on the one dimensional problem. A major theoretical challenge arises when the full two dimensional (transverse) problem coupled with a finite material response is considered. Both experimental (Gibbs at Optical Sciences Center and Boyd at Rochester) and preliminary theoretical work show that counter-propagating beam induced instabilities underpin complex nonlinear optical scattering processes. These scattering events can be associated with the formation of a complicated induced spatial grating at one-half the wavelength of the standing wave fields which, when mediated by a finite material response, can offer sufficient frequency bandwidth to generate a whole hierarchy of higher order Stokes/Anti-Stokes shifted waves. This area which offers many exciting challenges in spatio-temporal complexity, will require a heavy commitment of computational resources. Our existing time-dependent SBS code will form the basis for future computer modeling.

References

1. W. J. Firth, "Theory of Optical Bistability and Optical Memory," preprint, May (1988).

2. W. J. Firth and C. Paré, "Transverse Modulational Instabilities for Counterpropagating Beams in Kerr Media," Optics Letters **13** (12) December (1988).

3. C. Penman, W. J. Firth and C. Paré, "Transverse Modulational Instability of Counterpropagating Beams in a Kerr Medium Divided Into Thin Slices," preprint, (1989).

A.2. Counter-Propagating Beam Instabilities in Optical Fibers
(Aceves, Moloney, Wabnitz)

These complex dynamic interactions can be modeled and studied experimentally in optical fibers. The latter are essentially the optical analogues of large aspect ratio systems making them more amenable to analysis and computation. Both Aceves and Wabnitz have studied soliton structures in fibers with a periodically modulated linear refractive index extending some earlier analysis of Winful. We are currently investigating the role of group-velocity dispersion induced instabilities for counter-propagating fields. This boundary value problem promises to be extremely rich mathematically, admitting whole new classes of solutions, in addition to NLS type solitons. Experimental verification of our theoretical predictions will be possible through the experimental groups at Rome (Wabnitz) [1] and Heriot-Watt (R. G. Harrison). Preliminary experimental results at Heriot-Watt show a broad class of regular and chaotic light pulsations generated in the backward direction via stimulated Brillouin scattering. The origin of these instabilities is not understood.

Reference

*1. A. B. Aceves and S. Wabnitz, "Self-induced transparency solitons in nonlinear refractive periodic media," Phys. Lett. A. **141** 37 (1989).

A.3. Dynamics of Free-Running and Injection-Locked Laser Diode Arrays
(Indik, Jakobsen, Moloney)

This problem is based on a model of a phased array of diode lasers developed by Wang and Winful [1,2]. We have extended the model to include the case of an array driven by an external laser. For the case of no driving by external laser we have studied the stability of the well-known supermodes of the array as a function of the number of elements in the array. This work has proven that such devices are intrinsically unstable and grow more unstable as the number of elements increases.

We then moved on to studying the case when the laser array was driven by an external laser in addition to the current pumping. This was done in order to see if this driving could stabilize the array and thereby allow operation at higher field intensities. The supermodes were found to exist also in this case and their stability was studied. The supermodes were found to be stabilized by the driving.

In addition to the supermodes, we quite unexpectedly found a new class of solutions to the laser array model. These solutions are characterized by being potentially a much more powerful and useful solution than the supermodes described above. It is particularly interesting to note that the total energy output of these solutions increases as a fifth power of the number of lasing elements N in the array. This should be compared to the linear growth as a function of N for the supermodes. The linear stability analyses was done numerically and the solutions were found to be linearly unstable. A closer investigation found however, that the solution has a large strongly attracting manifold and a small and weakly repelling unstable manifold. This means essentially that the solutions upon perturbation first quickly attract and then on a much longer time scale go unstable. Extensive numerical simulations of the array have given us a good understanding of its long-time behavior. There is a new solution that seems to be attracting all initial states. This solution has the same fifth power dependence on N as the solution just described but has in addition, a certain essentially time independent phase profile. This phase profile makes the solution less useful for applications but fifth power dependence on N is intriguing. We are now in the process of trying to understand the nature of the instability better in order to be able to control it.

Pittsburgh Supercomputing Center (PSC) resources are an indispensable part of this work and we are very grateful for their support [3].

References

*1. S. S. Wang and H. G. Winful, "Stability and Phase Locking in Coupled Semiconductor Laser Arrays," preprint (1988).

*2. S. S. Wang and H. G. Winful, Appl. Phys. Lett. **52** 1774 (1988).

*3. E. J. Doedel, AUTO User Manual, Version September 1984.

A.4. Diffraction/Diffusion Mediated Instabilities in Self-focusing/Defocusing Nonlinear Amplifying Media
(Indik, Jakobsen, Moloney, Newell)

Diffraction of light and diffusion of the nonlinear excitation have been known for some time to profoundly influence the nonlinear response of passive optical feedback systems [1]. Transverse spatio-temporal instabilities in nonlinear amplifying media when mediated by diffraction and diffusion processes, form the basis for investigating the dynamics of broad stripe semiconductor lasers. Such lasers suffer the inevitable problem of multi-transverse mode instabilities which are further complicated by carrier diffusion within the wide gain region. Our preliminary analysis of the model describing this problem has led to the identification of new classes of transverse standing/traveling wave instabilities under both self-focusing and self-defocusing conditions. The latter instabilities exist only in the presence of a finite material response. Diffusion can shift instability bands in k-space or remove them altogether. We show moreover that the adiabatic elimination of the polarization variable from the Maxwell-Bloch equations significantly alters the nature of the instability growth curve. Future work will involve gaining an understanding of the self-defocusing traveling wave instability and extending our computation and analysis to the investigation of chaotic pattern dynamics in two dimensions.

Reference

*1. L. A. Lugiato, L. M. Narducci and C. Oldano, "Cooperative frequency locking and stationary spatial structures in lasers," J. Opt. Soc. Am. B **5** No. 5, May (1988).

A.5. Coupled-Wave Interactions in Extended Media
(Indik, McLaughlin, Moloney, Newell)

This project is aimed at developing a suite of computer codes capable of solving a variety of problems involving the interaction of two or more laser beams in extended media. The main focus of the problem has been the development of a time dependent three dimensional code to handle the complicated problem of backward Stimulated Brillouin Scattering (SBS). This code is running well at this time. We are now extending the code to include a third, acoustic field in the simulation, and improving the efficiency of the code by modifications to the algorithm.

This code is currently being used to test the models for SBS that seem to offer a possibility of explaining the Optical phase conjugation effects that have been observed. We have been limiting most of the use of this code to two dimensional problems to conserve computer resources.

We have continued our efforts to improve the efficiency of the code that we use to solve the SBS problem and other counter-propagating beam problems. We are incorporating suggestions from J. Hyman (Los Alamos National Laboratory) into the code to take advantage of an improved trapezoid rule algorithm as well as a scheme to increase the size of the propagation step we can use in our split step algorithms by integrating the equations for the first time derivative of our propagation equations. This should allow us the take advantage of the fact that the systems we are studying tend to converge to a nearly steady state after the initial transient behaviors have passed. The algorithms more typically used, do not allow one to take advantage of this behavior, since the individual steps of the split step are not at rest.

As before, all code is fully vectorized and we are testing codes on the Cray-2 at Kirtland. As mentioned above, we are also modifying the code to include the material response.

A.5.a. Theoretical Study of Optical Phase Conjugation in Stimulated Brillouin Scattering (Indik, McLaughlin, Newell)

As part of our study of the phenomena associated with Stimulated Brillouin Scattering, we have been doing some analytical explorations to try to understand how optical phase conjugation arises. We have taken the work of Hu et al. [1] and tried to reformulate it in such a way that we can understand when the model they derive will apply, and estimate the error terms arising from the simplifications in that model. In [1], a k-space analyses of the steady state equations describing stimulated Brillouin scattering is used in which terms that are not phase matched are ignored, and a certain additional term is inserted. Once these simplifications have been made, the authors show that the new equations can be reduced to a set of four ode's (from the original two pde's), which have coefficients that depend on the boundary conditions of the original problem. Once the reduction to ode's has been made, the problem of numerically integrating the SBS problem is vastly simplified. We have reformulated the results of the k-space analyses into equations in the original space variables which involve some transverse integrals. It is clear on inspecting these equations why the phase conjugation effect should arise. Moreover, from this form of the equations, we have been able to reduce the problem to a system of two ode's in physical quantities (reflectivity and fidelity) that are measured in the experiments.

We are currently engaged in the attempt to derive the above-mentioned pde's involving transverse integrals from the original equations, keeping good control of the errors and requirements for the various estimates. It is our hope that we will be able to derive these equations rigorously, and be able to predict in what regimes they will apply, and therefore, under what conditions optical phase conjugation can be expected to arise in Stimulated Brillouin Scattering [2,3,4].

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A.6. Nonlinear Optical Switching at Multiple Interfaces
(Aceves, Adachihara, Moloney, Newell, Varatharajah, Wright)

This highly successful ongoing project, catalyzed by our equivalent particle theory, has naturally evolved to include material diffusion, finite material response and pulsed spatial switching in multi-layered nonlinear wave guiding media. Recent experiments by a Bell Labs group have demonstrated the existence of spatial solitons propagating as 75 femto-second pulses in slabs of doped glass. This affords the experimental possibility of realizing the many nonlinear optical switching effects predicted by our equivalent particle theory. The extension of the theory to include nonlocal effects in space (diffusion) and time (retardation) as well as square and Gaussian pulse (in time) switching has allowed us to quantify various pulse stripping effects in single interfaces, directional coupler and Mach-Zehnder interferometer structures. Toward this end the equivalent particle theory has proved to be an invaluable design tool.

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A.7. Externally Encoded and Spontaneous Pattern Formation in a Nonlinear Optical Ring Cavity
(Adachihara, Lizarraga, Moloney, McLaughlin, Newell, Wenden)

Our ongoing research activity in this area has now evolved to a careful study of the underlying mechanism for complex pattern evolution across the two dimensional transverse cross-section of a laser beam circulating in a ring cavity. We are comparing a mean-field model with the full infinite dimensional map and our preliminary results with the former suggest that many of the dynamic features are essentially identical. This is extremely encouraging as the mean field model is essentially a two dimensional-HLS type problem with external forcing and damping, and this should be amenable to a detailed analytic study. The outstanding questions that remain are to understand the saturated filament interaction leading to collapse and the modulational instability of flat-topped solitary wave states. We are currently investigating the role of filament density in establishing complexity of the underlying patterns. The problem has been extended to include finite material response in one dimension. Work is currently underway on the linear stability analysis of the delay-differential problem describing the finite material response case. Numerical simulations suggest that the solitary wave structures are extremely robust to delay effects and indeed promise an even richer spatio-temporal pattern evolution.

Dark solitary waves can be encoded across the switched-on portion of the beam for self-defocusing nonlinearities. This is achieved by encoding a phase modulation across the external pump beam. The hard edge provided by the bistable response of the ring cavity provides a natural aperture with which to confine the defocusing beam. The dark furrows encoded across the beam can be removed and rewritten by adjusting the phase modulation on the pump. One interesting preliminary observation is that the so called gray solitons of the NLS problem, which must have finite transverse velocities, acquires zero velocity under external pumping and dissipation.

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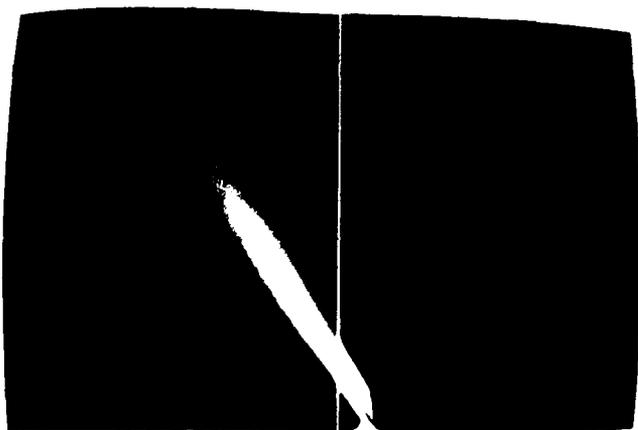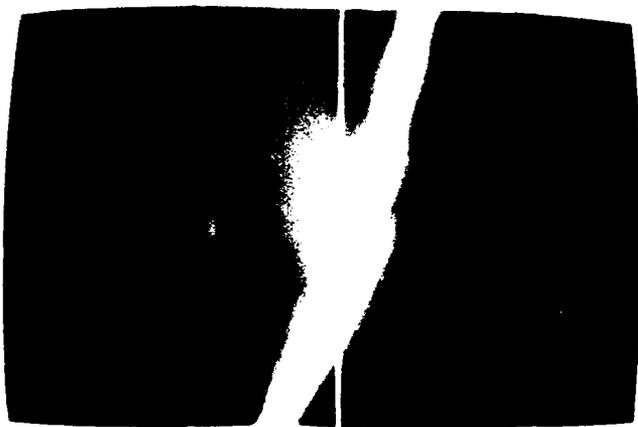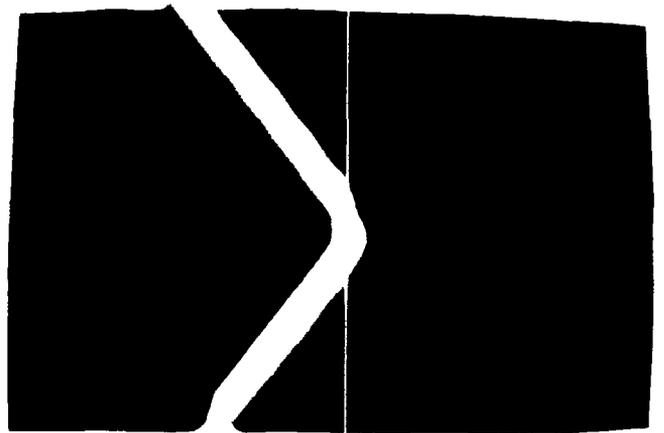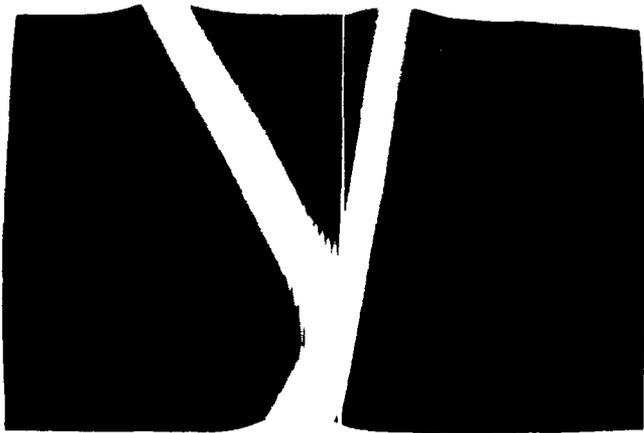
B. TURBULENCE AND THE NATURE OF SPATIO-TEMPORAL COMPLEXITY IN PDE'S

(Bayly, Ercolani, Levermore, McLaughlin, Newell, Passot, Pomeau, Rand, Rubenchik, Zakharov)

Unstable solutions, traditionally dismissed because they are not directly observable, can actually play a fundamental role in organizing the phase space of dynamical systems. The importance of this role has been realized in the theory of finite dimensional systems for some time. For example, orbits homoclinic to unstable fixed points can be used to establish the existence of irregular temporal behavior through the construction of a "horseshoe." However, the role of unstable solutions for pde's is just beginning to be appreciated. For pde's, an instability of a particular spatial pattern can produce more complex spatial structures. This production process can continue, generating even more interesting spatial structures and lead to the onset of both spatial and temporal turbulence. Even more dramatic is the role of singularities and collapse structures of the "Euler" (nonviscous) part of the governing equations because these structures can dominate transport properties in both real and wavenumber space.

We now describe some of these projects individually, attributing each to those scientists doing most of the work. We do emphasize that each project is but one part of the entire body of work, which is continually being discussed in its entirety throughout our Center. Each project benefits immensely from this open interactive environment and from scientific input from a variety of sources. Certainly visitors to the Center have played, and will continue to play, an important role. For example, in the area of instabilities and singularities, we mention in particular Volodja Zakharov, Sacha Rubenchik, David Rand, Yves Pomeau, Ed Overman and Pierre Coullet.

References to works completed under Center support are listed at the end of each subsection. For brevity's sake, in this report we do not list the vast number of important contributions made by other colleagues. These are fully documented in the published works.

B.1. The Role of Collapse Structures in Nonlinear Physics
(Zakharov, Newell, Shvets, Dyachenko, Pushkarev, Indik, Jacobsen)

In 1972, Zakharov suggested that the transport of energy from large scales to small dissipative scales in plasmas (at which the wave energy is converted into heat energy of electrons) was due to singular or collapse events rather than four wave mixing processes. Since that time, this idea has gained credence and is now quite generally accepted. Although most of the theoretical work has been carried out on model equations (the Zakharov equations, the vector and scalar nonlinear Schrödinger equations), recent numerical experiments on the full set of governing kinetic equations has verified the dominance of the collapse events.

We are presently carrying out a series of numerical experiments to simulate turbulent solutions of

the NLS equation:

$$i\psi_t + \nabla^2\psi + |\psi|^s\psi = \psi^* e^{-2i\lambda t} + i\epsilon_1\beta|\psi|^n\psi + i\epsilon_2\gamma\psi$$

in the case where the product sd (s is the order of the nonlinearity, d the dimension) is greater than or equal to 4. The goal is to verify with care the idea that the transfer of energy through k space and/or the dissipation rate is controlled by the collapse events [1,2,3,4,5,6].

Specifically we will first check two theoretical predictions.

1. In the limit of small linear damping, collapse is the leading mechanism of energy dissipation.

2. The asymptotic properties of the pair correlation function are determined by the local structure near collapse points.

Confirming or rejecting these will be very significant for the theory of strong turbulence in optics and in plasmas. One of our goals is to verify the role of superstrong collapse, namely the creation of hot spots or black holes which remain turned on until they literally suck all available energy out of the field. These objects are theoretically possible in an optical context in which the medium is self-focusing and three dimensional with anomolous dispersion in the direction of wave propagation.

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B.2. Coherence and Chaos in Near-Integrable PDE's
(Ercolani, McLaughlin, Schober, Roitner)

Near integrable nonlinear wave equations are excellent prototypes for the study of chaotic attractors for partial differential equations (pde's), particulary when these attractors exhibit both spatial coherence and temporal chaos. Our studies are the first in pde's to express explicitly homoclinic orbits with rich spa:ial and temporal behavior, to provide a complete classification of the homoclinic manifolds for an integrable nonlinear wave equation, and to identify numerically the presence of these homoclinic structures in the chaotic attractor of a nearby perturbed wave equation. These homoclinic solutions reside on a "boundary" between types of waves with distinct spatial characteristics in that, in each neighborhood of a homoclinic orbit, there exist solutions of two distinct spatial types, for example, extended nonlinear photons and localized solitary waves. Our studies suggest that these homoclinic manifolds play fundamental roles both in the saturation of instabilities by the generation of increasingly complicated spatial structures and as sources of sensitivity which produce chaos in near integrable pde's. A more explicit description of our results follows.

Numerical studies [1] of the damped driven sine-Gordon equation have identified rich phenomena in its low dimensional chaotic attractors including the presence of simple localized spatial (breather) excitations, the generation of more complicated spatial structures, and a competition between these spatial patterns which results in an intermittent, chaotic evolution. Theoretical studies [2] of the nearby integrable sine-Gordon equation have identified exact solutions with nontrivial spatial and temporal structures which are unstable toward solutions with still more complicated structure. These instabilities are in one-to-one correspondence with homoclinic manifolds of the completely integrable pde.

Direct numerical measurements [3] using the inverse spectral transform have established that, as the phase point evolves along a chaotic attractor, these homoclinic manifolds of the nearby integrable sine Gordon equation are frequently (and apparently irregularly) crossed. Thus, these studies have produced natural candidates for sources of chaos. Further theoretical studies [4] have constructed exact solutions of the driven damped sine-Gordon equation. Theoretical stability considerations classify these solutions as (i) stable, (ii) metastable (low dimensional unstable manifolds with small $- O(\epsilon)$, where ϵ is the perturbation parameter - growth rates), and (iii) unstable - with large $O(1)$ growth rates. The unstable states arise from deformations of those exact sine-Gordon solutions which are unstable in the absence of external perturbations, and, typically, they have simple spatial structure. The simplest such unstable state is x independent and is unstable to long wavelength perturbations (the classical Benjamin-Feir instability). The stable states tend to have a richer spatial structure. For example, one such state results from the deformation of a sine-Gordon breather whose temporal frequency is that of the driver. As the stress is increased, this stable spatially localized state becomes metastable, with a two dimensional unstable manifold and small $- O(\epsilon)$ - growth rates. A substantial second harmonic ($\cos 2kx$) is present in the unstable manifold [4].

The next theoretical step begins by viewing the chaotic attractor as predominantly composed of metastable states and homoclinic structures. The pde phase point will hover near one metastable state, slowly leave its neighborhood along and unstable direction, approach an unstable state along its stable manifold, rapidly fly past this unstable state, approach another metastable state, etc. This hovering near a metastable state, followed by flying rapidly past an unstable state toward another metastable state, describes the motion on the chaotic attractor. Several approaches can now be used to convert this picture into a mathematical description of the attractor for the pde. Currently, we are developing Melnikov calculations for pde's which are based on these homoclinic orbits [5], seeking low dimensioned approximate representations of the dynamics [6,7], and constructing an intuitive stochastic model of the chaotic motion. Recent work on Melnikov methods for near-integrable pde's is in collaboration with S. Wiggins (California Institute of Technology).

Very recent developments are:

B.2.a. Local Inertial Manifolds
(Ercolani, McLaughlin, Roitner)

With Arizona graduate student Heinz Roitner, Ercolani and McLaughlin have shown that for one dimensional periodic Schrödinger operators, the L^2-distance between potentials and the corresponding gap distance associated to the inverse spectral transform are <u>not</u> globally equivalent. However, they are equivalent on any bounded set in L^2. This work has been submitted to the Journal of Differential Equations.

We are applying this result to the analysis of a Kuramoto-Sivasinsky perturbation of KdV whose goal is to illustrate the construction of local inertial manifolds for nearly conservative PDE.

B.2.b. Exactly Solvable Models for the Propagation of Oscillations
(Ercolani, Wright)

With a graduate student, Otis Wright, Ercolani has shown how to explicitly solve the KdV modulation equations. This is being used to construct the non-local realization of the Young measure for the zero-dispersion limit of KdV.

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B.3. Local Inertial Manifolds and Adaptive Basis Schemes for Dynamical Systems (Broomhead, Indik, Newell, Rand)

We have suggested recently and developed a method for following the dynamics of systems whose long-time behavior is confined to an attractor or invariant manifold A of potentially large dimension. The idea is to embed A in a set of local coverings. The dynamics of the phase point P on A in each local ball is then approximated by the dynamics of its projections into the local tangent space. Optimal coordinates in each local patch are chosen by a local version of a singular value decomposition (SVD) analysis which picks out the principal axes of inertia of a data set. Because the basis is continually updated, it is natural to call the procedure an *adaptive basis method*. The advantages of the method are: (i) The choice of the local coordinate system in the local tangent space of A is dictated by the dynamics of the system being investigated and can therefore reflect the importance of natural nonlinear structures which occur locally but which could not be used as part of a global basis. (ii) The number of important or active local degrees of freedom is clearly defined by the algorithm and will usually be much lower than the number of coordinates in the local embedding space and certainly considerably fewer than the number which would be required to provide a global embedding of A. (iii) While the local coordinates indicate which nonlinear structures are important there, the transition matrices which glue the coordinate patches together carry information about the global geometry of A. (iv) The method also suggests a useful algorithm for the numerical integration of complicated spatially-extended equation systems, by first using crude integration schemes to generate data from which optimal local and sometimes global Galerkin bases are chosen.

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B.4. Turbulence, Hydrodynamic Stability Theory, and Dynamos (Bayly)

Research over the last decade has elucidated the basic mechanisms operating in the initial stages of turbulent transition in shear flows. The currently accepted picture is that the initial instability of a unidirectional (or otherwise essentially one-dimensional) flow is a two-dimensional wave of rather particular structure and scale. If no other disturbances are introduced, the wave typically equilibrates in a finite-amplitude two-dimensional structure. This wave flow is generally unstable to three-dimensional instabilities. The three-dimensional secondary instabilities occur on a wide range of length scales, in contrast to the primary two-dimensional instability. Full numerical simulations indicate that the secondary instabilities do not equilibrate but develop into fully turbulent shear flow.

The two-year report (Spring 1989) discussed the problem of instabilities in time-dependent flows containing finite-amplitude waves. The immediate precursor of this work was Patera and Amon's [1] study of flows in grooved channels, which was intended to model heat transfer above integrated-circuit boards. This work began by isolating the time-dependent elliptical vortex at the center of a travelling primary-instability wave. While apparently a simple generalization of the steady elliptical flow problem, this problem requires much more sophisticated mathematical machinery. The pulsating ellipse problem turned out to be isomorphic to the spectral theory of a quasi-periodic Schrödinger operator. Exploiting recent advances in the Schrödinger spectral theory, largely due to Johnson and Moser (e.g. [2]), we have been able to characterize the nature of the unstable modes and even verify the theory by numerical simulation. This work is in preparation.

A necessary part of the broadband instability theory is to develop some kind of nonlinear theory for the interaction of modes on different scales. Very little work has been done in this area. A suitable framework is gradually developing in which to examine various effects. These include weak nonlinearity, modes in confined geometries, and homogeneous but non-isotropic turbulence driven by modes in a particular cone in wavenumber space. This problem is still in preliminary stages.

Dynamo theory refers to the effort to understand the origins of the Earth's magnetic field, and that of the Sun and other large astrophysical bodies. The magnetic field in a highly conducting fluid obeys an equation identical in form to the vorticity in a high Reynolds number flow, or the equation representing stretching of a field of material line elements embedded in the fluid. Since Lagrangian chaos in a flowfield results in material line elements being stretched exponentially fast, there is likely to be a close connection between Lagrangian-chaotic flows and flows supporting robust magnetic field amplification.

The highly-conducting dynamo problem has been studied for several years now, in collaboration with S. Childress of the Courant Institute. We have demonstrated that simple chaotic flows are capable of fast dynamo action, and that such behavior persists in more general flows. Comprehensive investigations of dynamo action in nontrivial three-dimensional flows are being undertaken by Bayly in collaboration with S. Childress and R. B. Pelz (Rutgers). Pelz and Bayly are beginning a massive computational study of magnetic field generation on a HYPERCUBE parallel processor, whose architecture allows numerical investigations to be pursued at much higher resolution than has been possible on previous machines. Such computations will also be implemented on the Connection Machine parallel processor.

Bayly's work with Childress concentrates on deterministic models. A. Gilbert (of Cambridge, England and Nice, France) and Bayly have just completed a study of dynamo action in completely random flows. Developing an idea from work of Dittrich *et al.* [3], we investigated an extremely simple class of random flows that exhibit a wide range of interesting effects. In particular, our work has greatly simplified much of the confusion regarding the role of non-reflection-invariance in magnetic field generation.

Very recently, Bayly has made some theoretical progress on relating the numerical computations with the mathematical problem in the singular limit of infinite conductivity. The singular limit precludes any

direct connection with finite resolution simulations. However, it may be observed that the operations of numerical discretization and physical evolution almost commute, in a certain sense. This almost-commutation provides a mechanism for estimating the differences between the numerics and the true problem that is unnattainable otherwise. This approach, which appears to be new, offers promise for numerical attacks on other singular operators arising in fluid dynamics and dynamical systems.

A new project involving magnetism and fluid dynamics is in its early stages. Bayly is collaborating with W. Tam of the Physics Department on the dynamics of ferromagnetic fluids in oscillating magnetic fields. Such fluids are already used in technological applications, but their basic dynamics have been relatively lightly studied. The current investigation of nonlinear oscillations in a confined geometry is one of the first studies of a truly time-dependent ferrofluid phenomenon.

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B.5. Turbulence in Compressible Flows and the Incompressible Limit of the Navier-Stokes Equations
(Bayly, Jokipii,Levermore, Passot)

In this work (in collaboration with B. Bayly, J. Jokipii, D. Levermore and T. Passot), our interest has been focused on the study of different flow regimes of the compressible Navier-Stokes equation in the small Mach number limit [1]. The motivation is to provide a possible explanation to the observation made in the interstellar medium of the density fluctuations spectrum. This latter obeys a Kolmogorov-type scaling law ($< \delta\rho\delta\rho' >= k^{-5/3}$) over a large number of decades; this cannot be understood if it is assumed that the flow is in the "usual" quasi-incompressible regime where the density fluctuations are correlated to the pressure fluctuations which themselves scale as $k^{-7/3}$. We show on the contrary that, without invoking the influence of magnetic fields, it is possible to interpret the observed scaling law when another type of quasi-incompressible limit is reached where the density acts, at the dominant order, as a passively advected quantity. The entropy fluctuations play a dominant role in allowing a Boussinesq-type of equilibrium between the temperature fluctuations and the density fluctuations which then can be of a larger order of magnitude compared to the Mach number M, than in the usual incompressible limit where $\frac{\delta\rho}{\rho} = M^2$. We have used a direct numerical integration of the two-dimensional compressible

Navier-Stokes equations with a perfect gas equation of state to study the generation and the stability of such regimes with different types of forcing. It has been shown that a spatio-temporal distribution of heating and cooling provides an efficient and natural way of settling the fluid into such regimes (as also would a forcing provided by boundary conditions as in the Rayleigh-Bénard experiment).

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B.6. The Role of Defects in Creating Strong Turbulence
(Coullet, Lega)

A two dimensional pattern may be described by means of an order parameter, which measures the quantity of order in this structure. Defects are objects which break this order, and are, in that sense, elements of disorder. For instance, elementary defects of a two dimensional structure are points where the order parameter vanishes. Such objects appear in general because of initial conditions or edges, but they become fully interesting for the disorganization of the system as soon as they are produced spontaneously. We have shown in a previous work that a large scale instability of a two dimensional spatially extended pattern eventually leads to the creation of defects [1,2,3]. Because of their motion, the latter are then responsible for a loss of correlations in the system. The resulting state, which is a first stage on the way to disorder, has been termed defect-mediated turbulence. We have also shown that further away from the instability threshold, defect-mediated turbulence merges into amplitude turbulence, which is characterized by a large number of excited amplitude modes for the order parameter. The structure is then completely broken, and the field possesses numerous zeroes, which are reminiscent of defects in defect-mediated turbulence. In this limit, the dynamics of the order parameter can be related to that given by the two dimensional nonlinear Schrödinger equation, which is known to produce self-focusing in the considered regime of parameters. Hence, taking account of the presence of defects in the field could be a way of understanding strong turbulence.

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B.7. Three Dimensional Euler Equations
(Ercolani, Siggia)

With E. Siggia (Cornell University), Ercolani has recently determined the topological obstructions to the existence of global Clebsch variables for the three dimensional Euler equations. They are studying a Lagrangian formulation of Euler in these variables to analytically assess the possibility of singularity formation.

B.8. Analytical and Topological Studies of Singularity Formation in Euler's Equation
(Ercolani, Caflisch)

With R. Caflisch (UCLA), Ercolani has characterized the analytic structure of spatial singularities in a hyperbolic system with multi-valued initial data. The latter is a local model for singularity formation in the roll-up of a vortex sheet. They are currently extending their analysis to the non-local Birkhoff-Rott equation.

B.9. Shear and Turbulent Convection
(Zaleski)

S. Zaleski (with Levermore) is working on lattice gas methods for the simulation of hydrodynamics. He has proposed a lattice gas method for studying liquid-gas mixtures. Zaleski is also working on turbulent thermal convection. This involves theoretical work on the stability of boundary layers in thermal convection experiments, such as recent experiments at very high Rayleigh numbers performed in Chicago. These experiments yield several unexpected scaling laws. In particular the measured heat flux is much lower than predicted by simple theories. This may be explained by a similarity theory. Another tool for the study of thermal turbulence are numerical simulations of sheared boundary layers. These numerical simulations allow to reproduce in part characteristic effects in very high Rayleigh number experiments, such as the relative lowering of the heat flux with respect to marginal stability theory.

C. FLUIDS, FRONTS, STABILITY AND TRANSITION
(Chow, Levermore, Newell, Passot, Pomeau, Souli)

C.1. Convection Patterns
(Newell, Passot, Souli)

In a major breakthrough, we have derived and studied the phase and mean drift equations which describe the behavior of a convection pattern in large aspect-ratio containers [1,2,3]. The patterns typically consist of patches of slightly curved rolls separated by defects such as dislocations, grain boundaries and disclinations. We developed a macroscopic description of these patterns by averaging the Oberbeck-Boussinesq equations over the locally almost periodic structures using a method modelled on the ideas of Whitham who derived an analogue of the geometric optic for fully non-linear waves. The equations are derived arbitrarily far from onset and for any Prandtl number. The computation of the coefficients of the phase equation (which are functions of the Rayleigh number, the Prandtl number and the wavenumber) needed the construction of a complex algorithm and the utilization of tools new to this field.

Let us first briefly describe the steps involved in this derivation. As a starting point, we need to calculate for a given Rayleigh and Prandtl number, a fully nonlinear straight parallel roll solution of the Oberbeck-Boussinesq equations, and this for a set of values of the wavenumber k spanning the interval contained inside the marginal stability curve. This is done by using a Newton's method to solve algebraic equations obtained after projecting the PDE on a suitable truncated Galerkin basis. The solutions obtained are then interpolated in terms of k using cubic spline polynomials. Since the large-scale equations we are looking for are obtained as compatibility conditions in a multiple-scale expansion, the next step is to calcultate the linear operator obtained by linearizing the Oberbeck-Boussinesq equations around the straight roll solution. A major difficulty in this problem comes from the fact that the mean drift equation is obtained at the second order in the expansion. This necessitates the complete calculation of the first iterate, retaining in algebraic form its dependence on the wavevector $\vec{k}$. But since the linear operator is singular, we had to use a very robust method to evaluate the pseudo-inverse. We found out that a singular value decomposition is ideally suited for our purposes and that it also saves appreciable computer time. We had only to pay further attention to poor conditioning of the matrix in some cases.

As a first step, two kind of analysis were performed on this equation. The first one is a linearization around straight parallel rolls. It provided the borders of the nonlinear stability region that we found in exact agreement with the one of the Busse balloon concerning the long wavelength instabilities. The second one concerns the stability of target patterns. Because of the influence of sidewall boundaries, patches of circular rolls tend indeed to be the dominant pattern convection. The analysis of this simple model of circular rolls gives a natural way to calculate the selected wavenumber (which agrees closely with experiments) and gives rise to a new instability which appears to be important in initiating time dependence. We also predicted the Rayleigh numbers at which loss of spatial correlation due to global

defect nucleation will occur.

We are now investigating a generalization of this equation in order to describe defect nucleation. The equation obtained in [1] has the advantage of possibly describing a pattern of rolls pointing in any direction but has the drawback of losing its regularity when the wavenumber k exceeds by too much the boundary of the non-linear stability region. (This has been verified by direct numerical simulation of the nonlinear development of a skewed varicose instability.) A Ginzburg-Landau (or Newell-Whitehead) type of equation, which governs only the evolution of perturbations around straight parallel rolls, has however the property that it correctly describes the formation of defects. Our goal is to combine advantages of both formulations.

When the Rayleigh number is too high, we are left with a phase dynamics; the amplitude A of the rolls is indeed algebraically saved to the wavenumber k. This is wrong when A becomes small. The analysis must then be modified, and leads to two coupled partial differential equations for Θ and A. These equations are able to describe the formation of defects; they still posess singularities, but they are removable by a change of unknown: $w = A\exp(i\Theta)$. A matching between both descriptions has to be done if A is of order one or smaller.

The method is currently tested on simpler model equations for which the analysis is very simple but which contain all the ingredients of real problems, namely the Ginzburg-Landau equation. We are interested at describing the nucleation of a pair of dislocations when two-dimensional perturbations are superimposed on an Eckhaus unstable pattern.

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C.2. Nonlinear Dynamo
(De Young, Durney, Passot)

We studied the possiblity for the solar magnetic field to be generated in regions of weak buoyancy. Use has been made of numerical simulations of the EDQNM closure of the incompressible MHD equations, paying particular attention to the long time behaviour. In particular we were interested in the nonlinear saturation of the alpha effect. The main result was to prove that a solar dynamo located at the bottom of the convective region is indeed efficient to provide a magnetic field of the strength required by the observations, avoiding in the same way a lot of difficulties encountered when one trys to locate the generating region in the middle of the convective zone.

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C.3. Large Scale Instabilities in Tridimensional Compressible Flows
(Passot, Pouquet, Sulem)

It is known [2] that for a three-dimensional incompressible fluid, a large-scale perturbation superimposed on an anisotropic small-scale flow will be unstable if the basic flow lacks parity invariance and if the Galilean invariance is broken (either by the forcing mechanism or the boundaries). It had been proposed that in the case of a compressible flow, the assumption of anisotropy can be relaxed, and that the large-scale flow is subject to an helical instability of an alpha-dynamo type. Using a multiple-scale asymptotic analysis valid for small Reynolds number, we showed [1] that this is not the case and that the compressible effects do not alter very much the results of the incompressible case. We are planning to use direct numerical simultations in order to investigate the case of higher Reynolds numbers necessary in order for the compressibility to be able to play a significant role.

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C.4. Subgrid Scale Modeling in Two Dimensional MHD Turbulence
(Passot, Politano, Pouquet and Sulem)

In this study we have extended our work on hyperviscosity for compressible flows [1] to the case of two-dimensional homogeneous incompressible MHD flows [2]. We have been interested in comparing

the performance and validity of different formulations with the help of numerical simulations at high resolution. Numerical simulations without fudging at any scale of flows at Reynolds number of a few thousand served as references. The main result is that for such flows, at a comparable "effective" Reynolds number the large-scales are not affected by the precise functional form of the dissipation. We are thus planning to use such techniques in the more complex case of compressible MHD self-gravitating flows in order to study the influence of turbulence on gravitational collapse in presence of magnetic field.

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C.5. Caustics in Convection Patterns
(Pomeau)

Last year, some time was spent on Catastrophe theory. There is a well-known connection between this theory and the classical problem of formation of caustics in optical fields. Those caustics from wherever the assumptions of geometrical optics lose their validity because of singularities in the Huygens construction. Indeed, diffraction takes care of the local structure of the wave field near those singularities, and this is because one has a general solution of the Helmholtz equation in vacuum, owing to the linearity of this equation. The same Huygens construction allows to draw patterns of nonlinear waves with a fixed wavenumber, as for instance Rayleigh-Benard rolls of thermal convection. This leads quite naturally to the formation of caustics, when a generic focussing occurs. But then the diffraction dressing cannot be found as easily as for linear waves, as there is no general solution of the nonlinear wave equation. It has been shown, however, how to deal with this problem in the limit of the phase approximation, that is well-defined, and describes in a consistent way the nonlinear field in the neighborhood of a cusp singularity of this field.

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C.6. Evolution of Localized States and Fronts in Non-Gradient Flow Systems
(Pomeau, Jakobsen, Hakim)

Together with P. Jakobsen (University of Arizona) and V. Hakim (Laboratoire de Physique Statistique, Ecole Normale, Paris), we developed a comprehensive pertubative approach showing specific features of systems with neither a Hamiltonian nor a gradient flow dynamics: those systems are in some sense the most general. It was observed numerically that they can support finite amplitude and stable solitary waves, when a subcritical bifurcation takes place, and those waves have been observed as well in transition flows in lab experiments. We explained this and we made a connection with the two opposite limits (Hamiltonian and gradient) showing that on the Hamiltonian side those solitary waves merge with the famous solitons, but with a well-selected width, although on the gradient side they merge with front solutions at the Maxwell equilibrium point. By combining the two pictures as well as by making a reasonable extrapolation in between, we got a rather general understanding of a large class of possible time-asymptotic behaviors for those systems. This work calls for further studies in various directions.

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1. V. Hakim, P. Jakobsen and Y. Pomeau, "Fronts vs. Solitary Waves in Nonequilibrium Systems," to appear in Europhysics Letters.

C.7. Front Propagation
(Bernoff, Jones, Newell, Powell)

Currently, we are studying the propagation of fronts in Complex Ginzburg-Landau (CGL) type equations with quintic polynomial nonlinearities. These equations are partial differential equations (PDE) which describe the behavior of wavelike instabilities in a variety of systems, including hydrodynamics and laser optics. The quintic term models subcritical bifurcation from stability, which is a relevant physical behavior in convective systems. A front in such a system corresponds to the spread of a developed nonlinear behavior and is therefore a model for weak turbulent behavior. An argument was presented for understanding asymptotic front speeds using asymptotic spatial dependence. Observable front behavior occurs only when the asymptotic spatial behavior of a trajectory in the Gallilean ODE (Ordinary Differential Equation) corresponds to most unstable temporal behavior in the original PDE.

Integrable fronts have been found using the WTC (Weiss, Tabor and Carnevale) method. These special fronts are a consequence of the quintic nonlinearities and have no reflection in more standard CGL equations. Using the reasoning above, we can demonstrate that these fronts are predominant in physical parameter regimes. Analysis of the ODE phase space shows that the special class of WTC fronts have topological properties of strong heteroclinicity, which may help to explain their integrability. When fronts converge to this special class of solutions, the topology requires that they converge by leaving

behind a wave singularity. It is possible that this behavior is the one dimensional analogue of spiral waves in higher dimensions.

We have presented a Liapunov functional argument for front stability in particular circumstances. This argument allows us to understand the flow of solutions in function space and thus to understand some of the global behavior of fronts. Part of the current work is extending these arguments to more general versions of the CGL and to higher dimensions.

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C.8. The Theory of Compressible Fluids
(Levermore)

A new project was started this past year to study the mathematical theory of the compressible Navier-Stokes equations. The main questions being investigated are:

1. The possible global existence of weak solutions.

2. The convergence of such solutions to an incompressible limit.

Some partial results have already been obtained, the most significant being an existent result for the incompressible limit of a barotropic fluid.

C.9. Kinetic Theory of Fluids
(Levermore)

The past year has seen continued progress in this area.

1. The Leray energy inequality was proved for the incompressible limit for the global weak solution to the Boltzmann equations of R. DiPerna and P. L. Lions. This ODO shows that the weak solutions to the Navier-Stokes equations we had previously obtained for this limit are indeed the Leray solutions.

2. The "renormalization" needed in the DiPerna-Lions theory was weakened. It was shown that square root saturations can replace the linear saturations of the original theory. The implications of this fact need more investigation.

3. The control on the high velocity concentrations for the full incompressible Navier-Stokes limit of the DiPerna-Lions solutions was improved to within $(\log \epsilon)^2$ of the hypotheses needed to justify the limit.

Current Work:

1. Weakening or eliminating the hypotheses on high velocity concentrations for the full incompressible Navier-Stokes limit.

2. Better understanding the implications for the scaling assumptions embodied in our temporal regularity of the limit.

3. Consideration of other fluid dynamic limits, in particular, the validity of the compressible Navier-Stokes limit.

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C.10. Laminar-Turbulent Transition
(Chow)

The problem of laminar-turbulent transition is of great importance. Limited analytical progress can be made in the initial stages, where the developing and exciting technology of Laminar Flow Control (LFC) can be applied. Benney and Chow [1,2] had recently proposed a novel idea of wave - mean flow interaction which is very relevant and promising. By treating directly strong (not long wavelength modulation) three dimensional (3D) disturbance, we show that a 3D disturbance will:

1. Distort the flow on a much faster time scale than a corresponding 2D disturbance.

2. Induce much stronger mean flow than the conventional Ginzburg-Landau type theories.

Unfortunately, few exact solutions to such nonlinear wave mean flow interaction equations are known. The well known technique in applied mathematics, namely, linearization, will directly yield an analytical description of secondary instability. Up to now, we have achieved the following :

1. We investigate the 3D instability of the wake. 3D instability growth rates of a near and a far wake are computed. The results suggest the importance of 3D features in the far wake [5].

2. The wave - mean flow interaction problem of a compressible flow is examined. We study the 3D instabilities of a 'transonic jet' and a 'supersonic channel flow'. The paper is submitted to the Journal of Fluid Mechanics.

3. Some simple analytical properties and bounds on eigenvalues are found.

In addition, we investigate long waves in rotational flow [4,5]. Mean shear or depth dependent current is expected to have a profound effect on nonlinear surface waves in the ocean. It is an important problem but has not received much attention in the literature. We carry out a second order perturbation analysis for the solitary wave.

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D. COMPUTATIONAL SCIENCE
(Brio, Indik, LeMesurier, Levermore)

D.1. Computations in Optics
(Indik)

D.1.a. Phased Diode Laser Arrays

Phased laser diode arrays offer the possibility of providing high power spatially collimated light fields in miniaturized systems, making large high power conventional (CO_2) lasers redundant. Moreover, such arrays will form an important component as light sources in future anticipated massively parallel optical computing architectures. A number of challenging fundamental questions need to be addressed in this area. Of paramount importance is a knowledge of the dynamic stability of such laser arrays and the dependence of the latter on the various physical parameters. Arrays can be fabricated in one or two dimensions. For those arrays to function in an optical communications environment they will need to be individually externally modulated at very high frequencies (GHz).

P. Jakobsen, Indik, Moloney, Newell and H. Winful have studied the stability of one dimensional arrays of diode lasers. They have discovered an interesting new set of equilibrium solutions to the coupled mode equation model derived by Wang and Winful, and done extensive numerical investigations of the stability of the various solutions, as well as explorations of the bifurcation behavior of the model as the parameter corresponding to power is varied [1].

In addition, Jakobsen, Indik, Newell and Moloney are currently investigating the behavior of the one dimensional array, still using the coupled mode approximation, but with an additional term added to include the effect of an external driving field. The purpose of the external driving field is to stabilize the otherwise quite unstable phase locked modes of the system. We have discovered a number of new equilibrium solutions for the system, and have been able to find several stable modes, as well as a mode which is nearly stable (the instability is sufficiently slow that for pulsed modes the lasers would look stable), for which the power output increases as the fifth power of the number of lasers. This work is currently being written [2].

D.1.b. Coupled-Wave Interactions in Extended Nonlinear Optical Media

This project is aimed at developing a suite of computer codes capable of solving a variety of problems involving the interaction of two or more laser beams in extended media. The main focus of the problem has been the development of a time dependent 3D code to handle the complicated problem of backward Stimulated Brillioun Scattering (SBS). This code is running well at this time. We are now extending the code to include a third, acoustic field in the simulation, and improving the efficiency of the code by modifications to the algorithm.

This code is currently being used to test the models for SBS that seem to offer a possibility of

explaining the Optical phase conjugation effects that have been observed. We have been limiting most of the use of this code to two dimensional problems to conserve computer resources.

We have continued our efforts to improve the efficiency of the code that we use to solve the SBS problem and other counter-propagating beam problems. We are incorporating suggestions from J. Hyman into the code to take advantage of an improved trapezoid rule algorithm as well as a scheme to increase the size of the propagation step we can use in our split step algorithms by integrating the equations for the first time derivative of our propagation equations. This should allow us to take advantage of the fact that the systems we are studying tend to converge to a nearly steady state after there initial transient behaviors have passed. The algorithms more typically used, do not allow one to take advantage of this behavior, since the individual steps of the split step, are not at rest.

As before, all code is fully vectorized, an we are testing codes on the Cray-2 at Kirtland. As mentioned above we are also modifying the code to include the material response.

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D.2. Discontinuous Solutions of Hyperbolic Systems
(Brio)

In the last year we have continued to study questions related to the discontinuous solutions of the hyperbolic systems. In [1], we have started to study of a model problem, which we proposed in [2] in order to clarify discontinuous solutions for the magnetohydrodynamic and elastic equations. Currently, together with a graduate student Y.-F. Chen, we are extending this study to 3 × 3 system. In particular, we are studying travelling waves, admissibility criteria for shock waves, interaction of shock waves with small disturbance waves, and constuction of the solutions to the Riemann problem.

In [3], we have derived a canonical asymptotic equation which describes the transverse waves with certain isotropy property (rotational invariance) in the manner Burgers' equation describes the propagation of a single weakly nonlinear longitudinal wave. The derivation is done for a general class of rotationally invariant hyperbolic systems of conservation laws in one and several dimensions, and applications to various branches of continuum mechanics such as magnetohydrodynamics, elasticity and viscoelasticity are considered. In the future, we will extend this work by allowing systems with elliptic regions [4].

Currently, we are studying resulting asymptotic cubically nonlinear system in 1-D case with a diffusion term. In particular, viscous profiles and their relation to the new type of shock waves in magnetohydrodynamics and elasticity. Also, using a generalization of the above system by including a weak diffraction,

we will focus on the numerical study of the transverse stability of such shock waves and compare it to the 2-D Burgers' equation. The results will be presented in [5].

In [6], we have extended the notion of the solution to the inviscid Burgers' equation for a time larger than the breaking time by using its connection to the Kepler's problem, and following the known solution of the later problem. It allows us to obtain an explicit Fourier series solution for an arbitrary initial 2π-peridic initial data. As a by-product, we obtain a uniform asymptotic expansion of the solution near the breaking time, and suggests the behavior of the spectral coefficients to problems with discontinuities. Currently, we are studying the interpretation of the proposed regularization in terms of particle dynamics, motion of singularities on a complex plane, and its relation to the resonantly interacting waves such as Rayleigh surface waves.

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D.3. Lattice Gas Hydrodynamics
(Levermore)

The prospect that lattice gases can be used effectively to simulate real fluid dynamic behavior has lead to a renaissance in thinking about the relation between microscopic and macroscopic physics over the last four years. Within the last year we have seen continued development of the FCHC lattic gas for incompressible Navier-Stokes flow past obstacles. This gas lives on Z^3 with the state at each site coded by 24 bits corresponding to the 24 possible velocity states that can be occupied by particles. The collisional dynamics can be computed via a table look-up which requires at least 2^{23} entries of 24 bits, a table which fills a significant fraction of core on a CRAY-XMP.

Building on ideas of P. Rem and J. Somers, a new algorithm was developed for computing the collisional dynamics using compound applications of a single table containing 2^{16} entries of 16 bits. This significantly smaller table allowed the efficient implementation of the FCHC lattice gas on a Connection Machine CM-2 at Thinking Machines, Inc. in collaboration with B. Boghosian [1].

Another difficulty in lattice gas simulations arises from controlling the fluctuations in the spatial-temporal averages used to construct the macroscopic fields. This noise may be mitigated in two ways: one can make the lattice finer thus increasing the number of spatio-tempered sites averaged over; one can introduce an ensemble over which to average. Each of these methods presents difficulties; the first causes the time step represented by one cycle time to drop as the square of the refinement ratio, greatly increasing work, while the second method runs the risk of having the macroscopic dynamics of different elements of the ensemble to drift apart from each other. The technique of ensemble dynamics was developed to address the latter difficulty. Different implementations of theses ideas were applied to solving diffusion equations [2,3]. In each case, the build-up of unwanted correlations was carefully measured, clearly showing deviations from the lattice Boltzmann approximation. The best results were obtained when the "particles" moving between ensemble members did not respond to the particles contained within each member [3,4,5].

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D.4. Self-Focusing Phenomena in Lasers
(LeMesurier)

This work [1,2] has dealt principally with the initial value problem for the nonlinear Schrödinger equation:

$$i\frac{\partial \psi}{\partial z} + \sum_{k=1}^{k=d} \frac{\partial^2 \psi}{\partial x_k{}^2} + V(|\psi|^2)\psi = 0 \, , \, \psi|_{z=0} = \psi_0$$

The main cases studied have been:

1. The Kerr nonlinearity $V(I) = I$ with $d = 2$,

2. Variants of the Kerr nonlinearity that saturate at high beam power intensities I: $I/(\gamma + I)$ and $I - \gamma I^2$.

3. The Kerr nonlinearity with $d = 3$, a "supercritical" case.

4. $V = I^2$, $d = 2$, another supercritical case.

The first is a simple model of self-focusing of laser beams, the second refines this model in a way that eliminates singularities, and the third is a simple model related to Langmuir turbulence in plasmas, The fourth is a mathematical convenience: it solutions seem to have much the same pattern of behavior as the third equation, without the extra computational cost of working in three dimensions.

With the Kerr nonlinearity singular spikes can evolve (focusing), while with the saturating modification, one gets a train of intense narrow spikes (partial focusing). This leads to severe difficulties in numerical solution, and the main challenge has been the development of numerical methods that resolve the extremely fine spatial scales that develop in all coordinate directions.

The numerical solutions use a dynamically determined, single parameter, z dependent dilation rescaling of all variables to $\overline{x_k}$, $\overline{z}$, $\overline{\psi}$. This rescales transverse coordinates(x_k, longitudinal position z and field ψ by appropriate powers of a length scale for the developing focus so that the leading order terms and nonlinearity are unchanged, and only a lower order term is introduced. The length scale for rescaling evolves under an equation that holds the quantity $G = \int |\nabla \overline{\psi}|^2 d\overline{x}$ constant. G measures growth in ampliude and derivatives, and this rescaling eliminates any substantial growth in amplitudes and derivatives of the rescaled quantity. Any singularity is transformed to a cetain nice limiting behavior as $\overline{z}- > \infty$. Thus the transformed equations have well-behaved solutions, and can be solved numerically in a reasonably straightforward way. They are also useful, in somewhat modified forms, for analytical studies.

One difficulty is that the dilation rescaling should have a center at or very near the focusing center: otherwise the rescaled location of the singularity moves towards ·nfinity when a singularity occurs and far from the $\overline{x}$ origin with strong partial focusing.

This was initially guaranteed, along with faster numerical codes, by using cylindrically symmetric data. These simulations have been followed solutions to beam intensity growths by factors of up to 10^{10}

with the Kerr nonlinearity and 10^4 for saturating nonlinearities, revealing several interesting features near the foci. These include new conjectures as to the intensity growth rate and spatial structure near a focus, and evidence contradicting earlier conjectures about the decay of trains of multiple foci in the saturating case.

More recent work has removed the restriction of cylindrical symmetry, imposing reflection symmetry in each x_k axis to fix the focusing center at the origin. So far this his been applied to case 1. above. For a variety of initial data the solution evolves towards a cylindrically symmetric structure in the "inner region". That is the region near the singularity, whose size changes with the rescaling length scale: the region of constant dimensions in the x_k coordinates. These observations vindicate the previous cylindrically symmetric studies as relevant for this nonlinearity. They also suggest an extension of Weinstein's theorem [3] giving this convergence to symmetry in the special case of beams having exactly the minimum power needed for focusing to occur.

Current work in progress includes:

1. Checking whether convergence to singularity occurs in the other cases above. (This is considered quite likely *not* to hold in supercritical cases.)

2. Developing a new adaptive regridding strategy that will resolve multiple singularities and singularities without cylindrical limiting behavior, without prior knowledge of the location or form of the singularity. This will probably be based on keeping the beam power ($\int |\psi|^2$) nearly constant on grid cells, by analogy to the Lagrangian formulation for fluids.

3. Refining the physical modeling by adding such features as forward and back propagating waves and stimulated Brillouin scattering.

4. On the theoretical side, partial results have been attained in confirming the numerically inspired conjectures, and this effort will be continued.

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E. INTEGRABLE SYSTEMS AND GEOMETRY
(Ercolani, Flaschka, McLaughlin)

E.1. Gauge Field Constructions
(Ercolani)

With A. Sinha (Ohio State University), Ercolani has explicitly constructed the general solution of Nahm's equations which parametrize the space of k-monopoles. The latter constitute stationary finite energy point defect solutions for a three dimensional nonlinear electromagnetic theory and are higher dimensional analogues of two dimensional vortex states.

Ercolani, together with R. Montgomery (U. C. Berkeley), are currently using this construction to study monopole dynamics.

E.2. Construction of Constant Mean Curvature Surfaces
(Ercolani)

The structure equations for a doubly periodic constant mean curvature immersion in R^3 are equivalent to the elliptic sinh–Gordon equation and its associated z, $\bar{z}$ eigenvalue problems on a doubly periodic domain. Proof that solutions exist depends on showing that a highly transcendental set of arithmetic conditions can be satisfied. Recently Ercolani, together with H. Knörrer and E. Trubowitz (ETH, Zürich) have demonstrated this. The result has applications to the construction of stationary solutions to the hydrodynamic Euler equations in special geometries. More generally, they plan to develop these methods into a general tool with which to demonstrate existence for global problems in differential geometry.

E.3. Topological Classification of Integrable PDE
(Ercolani, McLaughlin)

With D. McLaughlin (Princeton University), we have modeled Fomenko's two degree of freedom integrable foliations in an infinite dimensional, spatially periodic soliton system. Specifically, the analyticity of the spectral transform provides a transparent description of the critical sets and, in their neighborhood, the twisting of the foliation. Numerical studies, by Arizona graduate student C. Schober, have shown that this topological signature has a crucial determining effect on how the integrable structure breaks up under perturbations. We are examining this analytically. The above work will appear in the proceedings of a 1989 MSRI summer workshop on symplectic geometry.

E.4. Painlevé Analysis of the Toda Lattice
(Flaschka)

The question is: how do the symmetries of an integrable system lead to regularizations of the solutions in the complex (time) domain? In the most accessible cases, all solutions are meromorphic, and one can

compactify the level varieties of the constants of motion. A particularly complete picture is available for the Toda equations associated to semisimple Lie algebras. Flaschka and Zeng [1] compute all the Laurent series solutions. Flaschka and Haine [2] study the geometry underlying the compactified Toda level varieties. Ercolani, Flaschka and Haine [3] show how the compactification is constructed, and they relate the geometry to dynamics.

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E.5. Momentum Mappings
(Flaschka)

The methods developed for the complex Toda lattice were applied to prove that the compactified *real* level surfaces are symplectically isomorphic, via a momentum mapping, to certain convex polyhedra [1]. As a byproduct, the convergence of the **QR** algorithm for tri-diagonal matrices is seen to follow from the asymptotics of a gradient flow in a natural Riemannian metric. Flaschka is currently trying to relate these ideas to recent developments in Poisson geometry, and then to extend them to general symmetric matrices. Flaschka and his student, M. Zou, are also investigating certain infinite–dimensional versions (involving loop groups) of these results.

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E.6. Nonlinear Poisson Structures
(Flaschka)

Flaschka's student, P. Damianou, in his thesis (May 1989), studied two problems in the geometry of Hamiltonian systems. First, he found an infinite family of compatible Poisson brackets for the Toda lattice; there is one bracket of degree n for each $n \geq 1$. His methods are new, and don't yet fit the familiar bi-hamiltonian theories. Damianou also computed a class of nonlinear Poisson brackets associated to the group of $n \times n$ matrices. The formulas for the brackets reflect singularities of sets of nilpotent matrices.

These are mostly experimental results, obtained by symbolic computation, and conceptual explanations still need to be supplied.

E.7. Topology of Level Surfaces
(Flaschka)

Flaschka [1] introduced a class of integrable systems whose level surfaces are neither tori nor cylinders nor planes, and which do not have the Painlevé property. He is currently analyzing other examples, to see what topological types are possible, and to understand the dynamics on those non-standard surfaces.

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F. RANDOM DISTRIBUTED SYSTEMS
(Kennedy, Newman)

F.1. Exactly Soluble 3D Random Surface Model
(Newman)

A major accomplishment was the discovery (jointly with D. Abraham) of a new exactly soluble three dimensional random surface model with a wetting transition. The inital announcement [1] appeared in Physical Review Letters, followed by a long paper [2] and a conference review [3]. Work is in progress on the relation between wetting and roughening in this model.

Numerical work on the trapping transition in two dimensional percolation (jointly with graduate student M. Pokorny and with D. Meiron) was completed [4]. It was shown that earlier claims (by other authors) of a new universality class were incorrect. Other work in percolation included the discovery of an intermediate "layered" phase for certain tree-like lattices [5] and the extension of those results to Ising models [6].

Significant progress was made on the longstanding open problem of proving continuity of the percolation phase transition for arbitrary spatial dimension. This was done for percolation in orthants [7] and half-spaces [8]. Work continues on the problem in a full space.

Motivated by issues related to Ising spin glasses, an unexpected property of domains in ordinary Ising ferromagnets was discovered [9]. Much current work (jointly with D. Stein, M. Aizenman and A. Gandolfi) concerns spin glasses and related percolation models.

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F.2. Ising Spin Systems
(Kennedy)

A rigorous study of the "majority rule" renormalization transformation for Ising spin systems is under way. The first step [1] was to develop finite volume criteria that insure that the infinite volume system is in the high temperature phase. The finite volume criteria can then be tested with the help of a computer. One of the key ingredients of the folklore about the majority rule transformation is that the constrained system that appears in the definition of the majority rule is in the high temperature phase even if the original system is at the critical point. Using the criteria of [1], I can now prove this happens in a particular example.

Numerical studies of quantum spin chains have addressed two different issues. The first study [2] is related to the Haldane's conjecture that there is a gap in the spectrum of the spin-1 Heisenberg chain. This numerical study was the first to consider chains with open rather than periodic boundary conditions, and found a different behavior for the low lying eigenvalues. One consequence of this behavior is a good estimate of the correlation length of the chain. A separate numerical study [3], in collaboration with D. Guo and S. Mazumdar (University of Arizona Physics Department), dealt with the existence of the spin-Peierls transition in quantum spin chains.

For classical spin systems in which the correlation functions decay exponentially there is a large amount of rigorous work on the power law correction to this exponential decay (Ornstein-Zernike decay). For the correlations in the ground state of a quantum spin model one can also ask what is this power law correction. The only examples in which anything rigorous could be said were examples that were exactly solvable. The proof has now been carried out in a model that does not admit an exact solution.[4] A similar question is to study the decay of the finite volume corrections to the ground state energy of quantum spin models. M. Pokorny (graduate student in Applied Mathematics) has begun work on this problem.

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G. DYNAMICAL SYSTEMS
(Liverani, Wojtkowski, Young)

A dynamical system is defined by a set of differential equations, or in the discrete time case, by a transformation of some state space. The goal of this subject is to understand the time evolution of physical processes. Wojtkowski and Young work with chaotic dynamical systems with exponential divergence of orbits and strong mixing properties. Their tools include methods from geometric analysis and ergodic theory. Wojtkowski specializes in conservative systems. He studies specific models from mechanics and has developed mathematical techniques for dealing with many of them. During the last two years he made a major breakthrough in the analysis of a system of n balls bouncing in a vertical tube with an elastic floor. This being a model for interacting particles, one is typically interested in the case when the number of balls is very large. Wojtkowski's work is the first successful analysis of this kind in which no restriction whatever is placed on the number of balls. Young works with general, qualitative theories that can be applied to a wider range of systems. Her main focus the last two years is on the role of noise. The question is the following: Given that physical measurements are never exact, and random fluctuations occur in ways beyond one's control, to what extent do these random forces shape or alter dynamical behaviors in the long run? Young obtained some answers to some of these questions.

G.1. Hamiltonian Dynamical Systems
(Wojtkowski)

In the last couple of years we concentrated on the problems of unstable (hyperbolic) and stable behavior in hamiltonian systems resulting in the following:

1. A remark on strong stability of linear Hamiltonian systems. (Journal of Differential Equations, **81** (2) 313-316).

   In this short paper we formulated a criterion for strong stability of the equilibrium in a linear hamiltonian system (or of the fixed point of a linear symplectic map). The simplest sufficient condition is positive definiteness of the (quadratic) hamiltonian. We introduce a series of quadratic first integrals and prove that strong stability is equivalent to positive definiteness of some linear combination of these first integrals.

2. A system of one dimensional balls with gravity.(To appear in Communications in Mathematical Physics).

   The system of one dimensional balls in an external field II.(To appear in Communications in Mathematical Physics).

   In the first paper we introduced a Hamiltonian system with arbitrary number of degrees of freedom (dimension) for which we can establish nonvanishing of at least one Lyapunov exponent almost

everywhere. It is a system of n particles in a line which fall down with constant acceleration towards a hard floor and collide elastically with each other. Particles can be considered point particles or they may have size (hard rods). We establish nonvanishing of Lyapunov exponents by introducing a quadratic form in the tangent bundle and checking that under the assumption of nonincreasing masses the quadratic form does not decrease and it increases on some tangent vectors. There is a similar situation in Sinai's gas of hard spheres. In the general case of arbitrary number of spheres [2,10] non-vanishing of only few Lyapunov exponents can be established. Although there is little doubt that the system has all exponents different from zero one encounters serious technical difficulties; see the paper about three balls by Krámli, Simányi and Szász, [7]. What one needs to prove to get nonvanishing of all exponents is first that every ball is connected by a chain of collisions with every other ball and secondly that certain conspiracies (too technical to formulate here) can occur only on orbits of total measure zero. For our system the former is taken care of automatically (all collisions that can occur do occur on all orbits) but we still cannot prove the latter.

In the second paper we modified the potential of the external field from $V(q) = q$ to $V(q)$ such that $V'(q) > 0$ and $V''(q) < 0$. These requirements allow in particular for the standard gravitational potential $V(q) = -1/q$. In such a system nonvanishing of all Lyapunov exponents can be established fairly easy under the usual assumption that the masses of the particles decrease as we go up. In this paper we refined the method of a Q-form for hamiltonian systems. We spell out the conditions which a quadratic form in the tangent bundle of the ambient phase space has to satisfy to project nicely on the subspace transversal to the flow. We believe that this formulation will find many other applications.

3. (with J. Cheng) Linear stability of a periodic orbit in the system of falling balls. (Preprint 1989).

In this joint paper with graduate student J. Cheng, we investigate the condition of decreasing masses in the system of falling balls with constant acceleration. Our goal was to check if this condition is necessary for mixing behavior in all of the phase space. Independent of the masses we find a periodic orbit and we establish its linear stability if the masses increase as we go up. Although we are unable to find the Birkhoff normal form and apply KAM theory rigorously our result is a very strong indication that in the case of increasing masses there are pockets of quasiperiodic motions in the phase space. It brings out the reason why so few systems are known where all of the phase space is in the mixing component.

4. Linearly stable orbits in 3 dimensional billiards. (To appear in Communications in Mathematical Physics).

We address here the problem of 3-dimensional billiard systems with convex boundaries. We construct a linearly stable periodic billiard orbit which reflects only in eight semispheres in $\mathbf{R}^3$. Given any distance l (not smaller than the diameter of the semispheres) we consider two parallel planes at this distance. We attach four semispheres to each of the planes (on the outside). The positions of the semispheres are the free parameters in our construction. We adjust them in such a way that there is a highly symmetric periodic billiard orbit which reflects only in the semispheres. We are still left with some free parameters and we show that in a certain range of these parameters the periodic orbit is linearly stable. Our construction shows that even with the simplest convex surface – the sphere, the presence of linearly stable orbits cannot be excluded by merely putting the convex pieces sufficiently far apart, which is the case for planar billiards [11].

Future Projects

1. The model of n falling particles in a line should be studied further. The first problem is to prove ergodicity. Recent work of Krámli, Simányi and Szász [8] clarifies the methods of Sinai and Chernov [2]. We hope that these methods can be adapted to the system of falling balls in an appropriate external field. In case of success this will be the first known system of interacting particles for which ergodicity can be established rigorously for arbitrary finite number of particles.

2. The measure theoretic entropy of the system (the sum of positive Lyapunov exponents) is positive. It is interesting to study its asymptotics as the number of particles increases to infinity. One would expect that if we keep the energy per particle fixed the measure theoretic entropy should grow linearly. If it does not it will require special explanation.

3. The periodic orbit in our system studied in [3] seems to be a good candidate for numerical study of Arnold diffusion. We found explicitly the Poincaré section map for this periodic orbit and we know fairly well when it is strongly stable (in linear approximation). The formulas are remarkably simple which allows for increase of the dimension (number of particles) so that hopefully the Arnold diffusion could be distinguished from numerical errors.

4. The methods we used in the study of the system on n falling balls do not allow the introduction of a ceiling which is quite natural from the point of view of statistical mechanics. More precisely we would like to study this system subject to additional constraint that the particles bounce off an elastic obstacle both at the bottom and at the top, i.e., they are confined to a finite box. Such a system is no longer self similar for different values of the total energy, which is the case in the system without the ceiling if the acceleration is constant. It is interesting to know if the introduction of the ceiling can indeed produce stable periodic orbits in the case of decreasing masses.

5. In a recent paper, Donnay and Liverani [4] found large classes of finite range potentials which produce nonvanishing of Lyapunov exponents in all of the phase space of a particle moving in the

planar central field. This is an improvement on the earlier work of Knauf [6] in which he applied Maupertuis principle to some problems of this kind and showed their equivalence to geodesic flows on surfaces with nonpositive curvature. They were unable to extend their results to three or more dimensions. On the other hand, it is well known that the Kepler problem in any dimension is equivalent for positive values of energy to the geodesic flow in the hyperbolic space [9],[1]. This indicates strongly that some sort of condition on the potential should guarantee the nonvanishing of Lyapunov exponents also in three and more dimensions. We hope the approach from [12] will give us such a condition.

6. Both in Sinai's gas of hard balls and in our model of falling balls the interaction of particles occurs via elastic collisions. We would like to find a smooth potential of interaction which would guarantee the strong mixing behavior in all of the phase space. We conjecture that in one dimension the standard gravitational potential ($V(r) = -1/r$) has this property. In particular all periodic solution of the colinear n-body problem are linearly unstable. Since this system is open (at any value of the energy) the conjectured hyperbolicity most probably implies that almost all solutions escape to infinity (one of the particles escapes to infinity).

7. It is interesting to study the asymptotic behavior of solutions in the system of falling balls in an external field which allows for escape to infinity (potential well of finite depth). For instance, is it true that for almost all orbits with k particles coming from infinity exactly k particles escape to infinity in the future? More generally one would like to understand better the consequences of local exponential instability in an open (noncompact) system.

8. The conjecture [13] that the system of falling balls with constant acceleration has all Lyapunov exponents different from zero remains unproven. Although some of the difficulties involved resemble those in [7] it may very well be a much simpler problem, where in particular the case of arbitrary number of particles could be treated.

9. Recently Gutkin [5] considered dynamics of hard balls with rotational degree of freedom. We plan to investigate Lyapunov exponents of this system. Based on our experience, [12] we hope that this additional feature of the dynamics will make it easier to establish nonvanishing of all Lyapunov exponents.

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G.2. Random Perturbations of Dynamical Systems
(Young)

By a randomly perturbed dynamical system, we mean a Markov process on the same state space. By *small* random perturbations of the diffeomorphism f, we refer to the fact that the transition probabilities give x are measures concentrated near the point $f(x)$.

The main interest is in two types of problems in this area. The first is how the presence of noise changes the dynamical characteristics of the system. One would expect the averaging effects of random perturbations to destroy accidental coincidences so that noisy systems would have nicer properties. This theme is explored in two papers by Young and Ledrappier [6,7].

The other question concerns statistical behaviors of zero noise limits. Would small random perturbations drastically alter the asymptotic distributions of points? (In order for this question to make sense, we must impose some conditions on our noise, such as some control on the densities of the transition

probabilities.) If a system is essentially impervious to small random perturbations, we will say that it is *stochastically stable*. The questions of which dynamical systems are stochastically stable and whether the typical system is stable or not are wide open. Y. Kifer proved in the early 1970's that Axiom A attractors are stable [5]. Later on, Young duplicated his result using a different approach [9]. Until a few years ago little was known beyond that.

Last year, in collaboration with F. Ledrappier, Young studied the effect of noise on the computation of Lyapunov exponents. We added some noise to each iteration of the dynamical system and compared the Lyapunov exponents of this noisy system to those of the noise free one. Our findings are that while deliberate attempts to sabotage computation results of this kind will usually succeed, genuinely random noise at low levels will almost surely not have drastic effects on the computation of exponents. These results are contained in [8].

Young also used the logistic maps as a model for study. This project was carried out jointly with M. Benedicks and the manuscript is in its final stages of preparation [2]. While logistic maps are very simple as dynamical systems, they are also known to be sensitive to parameter changes. That is, small changes in parameters can lead to different types of dynamics. Benedicks and Young proved that if our noise is sufficiently random and at a sufficiently low level, then most of these systems have very robust asyjmptotic distributions.

Young plans to look for more unified techniques that will enable her to obtain similar results for larger classes of dynamical systems.

G.2.a. Statistical Properties of Strange Attractors
(Young)

One of the major discoveries in this area in the late 1960's and early 1970's is that in the basin of attraction of an Axiom A attractor, Lebesgue almost every trajectory has the same asymptotic distribution. This discovery is due to Sinai, Bowen and Ruelle, and the distribution is known as the SBR measure. In principle at least, the notion of SBR measures can be extended to all strange attractors. (See e.g., [4].) In practice, however, it is extremely difficult to prove (or disprove) that these distributions exist for systems that are not Axiom A.

In collaboration with Benedicks, Young is studying this question for the so-called Henon maps. These maps have at the same time fascinated and frustrated mathematicians ever since their discovery by the physicist Henon. This is because they appear to be very simple, their dynamics clearly typify those of nonuniformly hyperbolic systems, and yet they have remained intractable for a long time. Recently Benedicks and Carleson showed that for a large set of parameters, these maps have attractors with very complex behavior [1]. Benedicks and Young believe they can prove that these attractors admit SBR measures, rendering them quite well understood after all from the statistical point of view [3].

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III. THE COMPUTATIONAL ENVIRONMENT FOR RESEARCH

Networked Computing

departmental ethernet

DOS Personal Computers

IRIS color workstation

SUN workstations

Math Department networked file servers

otherUniversity Departments

56kbs link to Cray2 at Kirtland AFB New Mexico

Appletalk/Ethernet protocol converter

Postscript Laser Printers

MacIntosh Workstations

Personal Computers

campus fiber-optic network

University of Arizona Computing Center

DEC VAX cluster

IBM 4381 mainframes

CYBER 175 mainframe

CONVEX C240

dial in phones

Bitnet mail services

to Westnet Supercomputer sites/NSFnet

to JVNC/NSF Supercomputer Centers

III. THE COMPUTATIONAL ENVIRONMENT FOR RESEARCH

In the last three years the availability of computational resources to members of the University of Arizona Mathematics Department and the Applied Mathematics Program has increased dramatically. This has had a dramatic effect on the research that has been undertaken within the department. The AFOSR grant to the Arizona Center For Mathematical Sciences, together with matching funds provided by the University, has allowed the Mathematics Department to buy the equipment necessary to let our computers communicate among themselves and with remote sites. We were initially able to buy the nucleus of a network of SUN workstations (4 stations, a server and the communications hardware to interconnect all of the SUNS) which has now expanded to include 22 SUNS and an IRIS super graphics workstation that was donated by Silicon Graphics Incorporated. We also have added a high speed link with the Cray-2 computer at Kirtland Air Force Base from the SUN network and we have been able to order an upgrade for the IRIS workstations that will dramatically increase its ability to produce real time graphics, and thus increase its usefulness in producing interactive graphics in combination with the Cray-2 or other supercomputers.

A wide variety of projects have benefited from the ready availability of time on various systems. The projects have ranged from Abstract Algebra and Number theory, to Fluid Mechanics and Nonlinear Optics to Neural Nets and Percolation. The computers that have been of primary use to researchers include the Department's network of 22 SUN graphics workstations, the Departmental Vax 750, the University's convex C240 mini-supercomputer and the NSF supercomputers in the centers at Princeton, San Diego and Pittsburgh. We have also been making good use of the Cray-2 at Kirtland Air Force Base. The SUN workstations are used for a variety of interactive jobs. Research programs involving small to moderate computations in Algebra, Optics and Fluids have been developed and run on the SUNs. In addition, the windowing on the SUNs make them an excellent environment for development and debugging larger programs destined to be run on a mainframe or a supercomputer. The SUNs also provide access to NSF net and Kirtland Air Force base. We use them to provide graphics for remote jobs running on larger computers. We will soon have a SUN 4 added to the network. That will provide us with mainframe power for intensive numerical and graphical applications.

Recently, an IRIS 4D super graphics workstation was donated to us from Silicon Graphics Incorporated. We plan to make this machine an important part of the training of graduate students in the computational sciences. The addition of a graphics machine capable of displaying real time three dimensional color animation means that systems that we could only examine at a few selected times can now be taken in dynamically. This is also a very important tool for studying and developing intuition about dynamical systems. The photographs at the beginning of Section II were taken on the IRIS.

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The University provides those faculty members with research grants access to a VAX 8650 mainframe. This mainframe machine has a wealth of software packages and libraries installed including MACSYMA and DISSPLA. It is frequently used for projects which are too big to run on the SUNs. Quite a few Optics and Fluid projects have been run there.

For projects that will not run on a workstation, we have access to the various national supercomputing centers via NSF net. Of course, a separate grant application for time is required, but we have the advantage of being able to develop programs and test prototypes on our local machines. In addition, we have the local graphics capability to analyze and display the data that is generated on the supercomputers. One of the most serious problems in making effective use of supercomputers is interpretation of output. The volume of data that the supercomputers can and do produce is staggering. Investigators who can glance over a couple of pages of numbers, scribble for awhile and understand the output from a problem that runs on a mini-computer, need a completely different approach with the reams of data produced from a supercomputer. The IRIS graphics workstation, the SUN workstations and, to a lesser extent, the graphics terminals that are available within the department make it possible to quickly generate and examine graphical data. In fact, the limiting factor is often the speed of transmission of the graphical data from the remote center to us.

We have started a collaboration with colleagues at Kirtland Air Force Base to study some problems in nonlinear optics. They have given us access to their Cray-2 supercomputer and we have recently set up a 56 K-baud high speed link. In the meantime, we have had telephone access which has allowed for some initial development of applications. Our limited ability to move data back and forth for analyses has been the bottleneck to date, but now that the high speed link is in place we have a tremendous amount of computing power intimately linked with our graphics workstations. We have been able to make use of the large memory and parallel architecture of this machine to study problems in time dependent, three dimensional, nonlinear optics. The combination of a high speed link to the Cray-2 and the graphics on the IRIS 4D should make it possible to understand the transient behavior of the optical equations.

As the preceding indicates, our computing environment is already very strong. We are continuing to expand that environment with connections to remote machines and relationships with other research organizations. There is a tremendous variety of research activity making use of the computational resources and we believe those resources have made a real difference in the amount and quality of research that has been produced.

IV. SPECIAL YEAR PROGRAMS

Over the past few years, the Department of Mathematics has organized Special Years in several areas. The purpose of these events is to bring to Arizona a mix of established and young mathematicians who will work together (in seminars, lecture series, joint projects) with our own faculty and students in order to explore in the broadest possible way frontiers of their chosen ara. In Spring 1986, there were two programs, one on Algebraic Geometry with an emphasis on Abelian varieties and another on Chaos and Turbulence. In 1987, our focus was on Computational Mathematics. In the Spring of 1988, in collaboration with the Complexity Center, the emphasis was Probability and Applications and brought together researchers interested in Statistical Mechanics, Image Processing, Random Media and Chaotic Dynamics. Other topics also under consideration are Geometry and Mathematical Models in Physiology.

The listings below are visiting colleagues who participated in each of these events.

Spring 1986
Chaos and Turbulence
A. Bernoff, Cambridge University
T. Bohr, University of Copenhagen
H. Brand, Wiezmann INstitute
M. Casdagli, La Jolla Institute
S. Chow,Michigan State University
P. Coullet, University of Nice
M. Cross, California Institute of Technology
P. Hohenberg, Bell Laboratories
J. Marsden, University of California-Berkeley
R.MacKay, University of Warwick
A.Mazor, Los Alamos National Laboratory
S.Newhouse, University of Maryland
B.Nicolaenko, Los Alamos National Laboratory
A. Pocheau, CEA, Saclay-Orsay
D. Rand, University of Warwick
E. Siggia, Cornell University
M. Tabor, Columbia University
C. Tresser, University of Nice

Algebraic Geometry
E. Arbarello, University of Rome
A. Ash, Ohio State University
Sir M. Atiyah,Oxford University
S. Beckmann, University of Pennsylvania
K. Coombes, University of Michigan
M. Cornalba, University of Pavia
R. Donagi,Northeastern University
D. Gieseker, University of California-Los Angeles
L. Haine,University of California
S.Shatz, University of Pennsylvania
C. Tracy, University of California-Davis
B. Van Geemen,Institute for Advanced Study

Fall 1986
Computational Mathematics
A.O.L. Atkin, University of Illinois-Chicago
A. Bernoff, Cambridge University
F.R. Beyl,Portland State University
R. Blecksmith,Northern Illinois University
J. Buchmann, University of Dusseldorf
J.P. Caputo, Grenoble
M. Casdagli, University of California-San Diego
J. Deutsch, Brown University
U. Hornung,Munich University
A. Iserles, Cambridge University
P. Jurs, Pennsylvania State University
H. Karzel, Munich Technological University
D.H. Lehmer, University of California-Berkeley
E.Lehmer, University of California-Berkeley
P. Lewis,Naval Postgraduate School
J. McKay, Concordia University
E. Overman, Ohio State University
M. Newman, Australian National University
M. Newman, University of California-Santa Barbara
H. Pahlings, RWTH Aachen
S. Perone, Livermore Laboratory
V. Pless, University of Illinois-Chicago
G. Simmons, Sandia Laboratories
N.J.A. Sloane, Bell Laboratories
O. Taussky, California Institute of Technology
J. Todd, California Institute of Technology
S. Wagstaff, Jr., Purdue University

Spring 1987
Computational Mathematics
C. Beattie, Virginia Polytechnic Institute
R. Eggleton, University of Newcastle
R. Guy,Univeristy of Calgary
M. Israeli, Technion, Haifa
J. Lagarias, Bell Laboratories
F. LeDrappier,University of Paris
A. Odlyzko, Bell Laboratories
J. Selfridge, Northern Illinois University
D. Shanks, University of Maryland
H.C. Williams, University of Manitoba

Spring 1988
Probability/Statistical Mechanics
D. Abraham, Oxford University
M. Aizenman, Courant Institute
G. Baker, Jr., Los Alamos National Laboratory
J. Bricmont, Princeton University
M. Denker, University of Heidelberg
S. Geman, Brown University
G.R. Grimmett, Unversity of Bristol
P. Hagen, Los Alamos National Laboratory
T. Kennedy, Princeton University
R.Maier, visiting, University of Arizona
T. Prost, FU Notre Dame Namur (Belgium)
E. Seiler, University of Munich
H. Spohn, Max Planck Institute, Munich
H. Zoladek, Warsaw University

Fall 1989
Biomathematics
M. Avellaneda, Courant University
P. Diamond, Universityof Queensland
B. Fittingof, Syracuse University
W. Getz, University of California-Berkeley
S. Grossberg, Boston University
V. Krinsky, Institute of Biological Physics, USSR
D. Lewis, University of Minnesota
B. Nicolaenko, Arizona State University
H. Papenfuss, Ruhr University, West Germany
D. Szasz, Hungarian Academy of Sciences
G. Wokowicz, McMaster University

Spring 1990
Nonlinear Optics and Turbulence
P. Coullet, University of Nice
A. Dyachenko, Institute of Cybernetics, USSR
I. Gabitov, Academy of Science, USSR
J. Lega, post-doc, University of Arizona
R.J. McKellar, Univsity of New Brunswick
Y. Pomeau, Ecole Normal Superieure
A. Pushkarev, Institute of Cybernetics, USSR
A.Rubencik, Novaibirsk,USSR
Z.-S. She, Princeton University
V. Shvets, Institute of Cybernetics, USSR
E. Siggia,Cornell University
V. Zakharov, Landau Institute, USSR
S. Zaleski, Paris

V. BIOGRAPHIES

A - FACULTY

BRUCE BAYLY, 29, Ph.D. 1986, Princeton University. Postdoctoral visiting member 1986-88 at Courant Institute of Mathematical Sciences; Assistant Professor, Mathematics, University of Arizona 1988-.
Research Interests: Kinematic and dynamical problems in three dimensional steady state flows.

MOYSEY BRIO, 37, Ph. D. 1984, University of California, Los Angeles. Assistant Research Physicist, Physics, UCLA, 1984-87; Assistant Professor, Mathematics, University of Arizona, 1987-.
Research Interests: Magnetohydrodynamics, fluid dynamics.

KWOK WING CHOW, 31, Ph.D. 1986, Massachusetts Institute of Technology. Post-Doctoral Instructor, 1986-88; Assistant Professor, Mathematics, University of Arizona 1988-.
Research Interests: Alternative approach to stability theories.

NICHOLAS M. ERCOLANI, 37, Ph.D. 1980, University of California at Berkeley. Assist. Professor, Ohio State Univ. 1980-1984; Assoc. Professor, Univ. of Arizona 1984-present. National Science Foundation Fellow 1984-87. Sloan Foundation Fellow 1987-88.
Research Interests: Algebraic geometry, complex function theory, spectral theory, integrable pde.

WILLAM J. FIRTH, 45, Ph.D. 1975. Visiting Researcher, Heidelberg University 1978-79; Senior Lecturer Heriot-Watt University, 1982-85 and Professor in Physics, Strathclyde University, 1985-.
Research Interests: Transverse diffusion and diffraction in optical bistability, optical memory arrays, instabilities in nonlinear optical systems, phase conjugation and four-wave mixing in Kerr media, bistability and instabilities in semiconductor laser amplifiers.

HERMANN FLASCHKA, 45, Ph.D. 1970, Massachusetts Institute of Technology. Professor, University of Arizona, 1980-; Visiting Professor, Research Institute for Mathematical Sciences, Kyoto, Japan.
Research Interests: Nonlinear wave motion, dynamical systems, algebraic geometry and Lie theory connected with dynamical systems.

BRENTON LE MESURIER, 31, Ph.D. 1986, New York University. Postdoctoral Research Associate, Rensselaer Polytechnic Institute 1985-87; Assistant Professor, University of Arizona 1987-.
Research Interests: Singularities in nonlinear partial differential equations.

CHARLES DAVID LEVERMORE, 38, Ph.D. 1982, Courant Institute, New York University. Mathematician Lawrence Livermore National Laboratory, 1982-1988; Associate Professor, Mathematics, University of Arizona 1988-.
Research Interests: Nonlinear partial differential equations, computational mathematics, cellular automata.

DAVID W. MC LAUGHLIN, 45, Ph.D. 1971, Indiana University. Chairman, Program in Applied Mathematics, University of Arizona 1986-; Co-director of Arizona (AFOSR) Center of Mathematical Sciences, 1986-; Associate Director (Acting), Center for the Study of Complex Systems, University of Arizona, 1988-.
Research Interests: Mathematical physics, nonlinear waves, theoretical nonlinear optics, singularities in nonlinear pde.

JEROME V. MOLONEY, 42, Ph.D. 1977, University of Western Ontario, Canada. Reader in Physics, Heriot-Watt, 1988-. Regular long-term visitor of Mathematics, University of Arizona, 1984-.
Research Interests: Nonlinear optics, stability and propagation of nonlinear waves in planar waveguides, transverse switching waves and solitary waves in optical bistability, instabilities and chaos in lasers.

ALAN C. NEWELL, 48, Ph. D. 1966, Massachusetts Institute of Technology. Chairman, Dept. Math. Comp. Sci., Clarkson Univ. 1971-79; Chairman, Appl. Math. Prog., Univ. of Arizona 1981-85; Chairman, Dept. of Math. 1985-. Guggenheim Fellow 1976-77. Humboldt Fellowship 1988-89. Member and Chairman (1987-88), NSF Advising Board on Math. Sciences (1986-89), Member, Board of Math. Sci. Nat. Acad. (1987-90).
Research Interests: Pattern Dynamics, Nonlinear Optics, Nonlinear Wave Propagation, Turbulence.

CHARLES M. NEWMAN, 44, Ph.D. 1971, Princeton University. Regular Faculty Positions: New York University 1971-73; Indiana University 1973-79; University of Arizona 1979-. Visiting Positions: NATO Postdoctoral Fellow, Technion 1975-76; Visiting Member, Courant Institute 1982; Lady Davis Visiting Professor, Hebrew University 1983. Sloan Fellow 1978-81; Guggenheim Fellow 1984-85.
Research Interests: Statistical mechanics, probability theory, mathematical biology.

YVES POMEAU, 48, Ph.D. 1967, Orsay University, France. Researcher Centre de la Recherche Scientifique from 1965-, and directeur since 1983. Corresponding member of the French Academy of Science in the division of Sciences Mechaniques. Consultant, Los Alamos National Laboratories.
Research Interests: Statistical physics, fluid mechanics, and dynamical systems.

DAVID RAND, 41, Ph.D. 1973, Southampton University, United Kingdom. Lecturer in Mathematics, 1978-80; Professor of Mathematics, Warwick University 1980-. Regular long-term visitor of Mathematics, University of Arizona, 1984-.
Research Interests: Dynamical systems, chaos and turbulence.

MACIEJ P. WOJTKOWSKI, 38, Ph.D. 1977, Moscow State University. Assistant Professor, Mathematics Department, University of Arizona, 1985-88; Associate Professor, University of Arizona 1988-; Sloan Fellow 1987-89.
Research Interests: Dynamical systems.

LAI-SANG YOUNG, 38, Ph.D. 1978, University of California at Berkeley; Assistant Professor, Northwestern University 1978-80; Visiting Lecturer, University of Warwick 1980-84; Assistant Professor, Michigan State University 1980-84; Associate Professor, Uiversity of Arizona, 1986-; NSF Visiting Professorship for Women in Science and Engineering 1982; fellowship at MSRI (Berkeley) 1983-84; Sloan Fellowship 1985-87.
Research Interests: Dynamical systems and smooth ergodic theory.

V. E. ZAKHAROV, 50, Ph.D. 1971, Novasibisk State University, USSR; Head, Plasma Theory Labs, Novasibisk Institute of Nuclear Physics, 1967-74; Landau Institute for Theoretical Phys, Moscow, 1974-; Visiting Professor, Mathematics Department, University of Arizona, 1990-.
Research Interests: He has been involved in weak turbulence, optics (he solved the problem of coherent-pulse propagation in an amplifier medium), solitons (he and Shabat were the first to solve the nonlinear Schrodinger equation), singularities, field, theory, characterization of integrable systems, strong turbulence (he was one of the authors of the collapse theory for Langmuir turbulence).

B - POST DOCTORALS AND STAFF

POST DOCTORAL

A. ACEVES, 32, Ph.D. 1988, Research Associate, Department of Mathematics, University of Arizona 1988-.
Research Interest: Nonlinear optics, nonlinear wave phenomena, numerical computing.

D. J. BARSKY, 29, Ph.D. 1986, Rutgers University, New Brunswick, New Jersey. Research Associate, Department of Mathematics, University of Arizona, 1987-.
Research Interest: Statistical mechanics, percolation and Ising models.

A. J. BERNOFF, 30, Ph.D. 1985, Applied Mathematics, Trinity College, University of Cambridge. Research Associate, Department of Mathematics, University of Arizona 1986-.
Research Interests: Convection, waves in fluids.

P. DAMIANOU, 36, Ph.D. 1989, Research Associate, Department of Mathematics, University of Arizona, 1989-.
Research Interests: Poisson structures.

W. M. HENRY, 28, Ph.D. 1988, Australian National University. Research Associate, Department of Mathematics, University of Arizona 1988-.
Research Interests: Optics.

J. LEGA, 26, Ph.D. 1989, Universite de Nice. Research Associate, Department of Mathematics, University of Arizona 1989-.
Research Interests: Physics of instabilities; numerical simulation of ODE's and PDE's; defects of macroscopic structure; Ginzburg-Landau approach of instabilities in macroscopic systems and pattern formation.

T. PASSOT, 30, Ph.D. 1987, Nice Observatory, France. Research Associate, University of Arizona 1988-.
Research Interest: Turbulence, convection patterns, compressible flows, self gravitation, numerical simulations, Painleve analysis.

M. SOULI, 34, Ph.D. 1984, University of Nice, France. Research Associate, University of Arizona 1988-.
Research Interests: Computational science, nonlinear optics, convection patterns.

STAFF

R. INDIK, 33, Ph.D. 1982, Princeton University. Assistant Professor, Brandeis University 1982-86. Computer Software Specialist, Mathematics Department, University of Arizona 1987-.
Research Interest: Nonlinear optics, number theory, algebraic geometry.

R. CONDON, 37, BA 1973, Harvard College. Computing Manager Department of Mathematics, University of Arizona 1986-.
Research Interest: Distributed processing systems, concurrent programming languages.

VI. GRADUATE STUDENT PROJECTS AND BIOGRAPHIES

A. ACEVES, B.S. 1981, Universidad Nacional Autonoma de Mexico; M.S. 1983, California Institute of Technology. Ph.D. 1988, University of Arizona. In the past 2 years, developed with J. V. Moloney and A. C. Newell a theory which describes the global reflection and transmission characteristics of beams propagating in nonlinear dielectric media with one or more interfaces. The main result is that the theory provides the nonlinear Snell's laws of reflection and transmission at an interface in a very simple way. These laws are obtained from an equivalent particle description of the beam.

R. ACEVES - B.S. 1981, Universidad Nacional Autonoma de Mexico. Ph.D. student in Applied Mathematics. Enrolled in 1984. Coherent pulse propagation in an inhomogeneously broadened medium is an exactly solvable model of wave propagation in a random medium in which an initial pulse decomposes into its soliton (coherent) and radiation (phonon) components. The latter is trapped in a localization distance; the former propagates without loss. It is our idea that this sort of decomposition is relevant to many systems in which there is a competition between the coherence of nonlinear wavepackets and the scattering of the random medium. We plan to exploit variations of the exactly solvable model. Advisor: A. C. Newell.

H. ADACHIHARA - B.S. 1983, Northern Michigan University. Marquette, Michigan. Ph.D. student in Applied Mathematics. Enrolled in 1983, Ph.D. 1988, University of Arizona. Worked on the circulation of a laser beam in a passive optical ring cavity. Found that the transverse beam profile undergoes a sequence of modulational instabilities leading to ring and filament structures that occur in a random fashion. Advisors: D. W. McLaughlin and A. C. Newell.

P. DAMIANOU - B.S. 1977, University of New Hampshire. M.A. 1988, University of California, Los Angeles, California. Ph.D. 1989, University of Arizona. Enrolled in 1985. Working on multiple Poisson structures for the Toda lattice, and on Poisson structures transverse to orbits in semi simple Lie algebras. Advisor: H. Flaschka.

L. FAHLBERG - B.S. 1975 and M.S. 1980, University of Kiril and Metodij, Yugoslavia. Ph.D. student in Mathematics. Enrolled in 1987. Started working on small random perturbations and asymptotic distributions for the Lorenz attractors. Advisor: L. Young.

P. JAKOBSEN - B.S. 1984, M.S. 1986, University of Tromso, Norway; Ph.D. student in Applied Mathematics. Enrolled in 1987. Working on study of stability, dynamics and bifurcation of externally driven and undriven arrays of diode lasers, as well as study of the linear stability of homogeneous solutions describing almost monochromatic plane waves propagating in a laser cavity. Advisor: A. C. Newell.

S. JIN - B.S. 1982 and M.S. 1986, Fudan University, Shanghai China. Ph.D. student in Applied Mathematics. Enrolled in 1987. Studying the semiclassical limit for solutions of nonlinear Schrodinger equations emphasizing the development and propagation of singularitis. Advisor: D. W. McLaughlin.

C. LIZARRAGA - B.S. 1978, Universidad de Sonora, Mexico; M.S. 1980 Universidad de Mexico. Ph.D. student in Applied Mathematics. Enrolled in 1985. Involved in research in propagation in a random medium, and localization problems in one dimension, and (principally) optical ring cavity behavior in the presence of a finite material relaxation time, with one transverse dimension. Advisor: A. C. Newell.

Y. LU - B.S. 1982, Fudan University, People's Republic of China. Ph.D. student in Applied Mathematics. Enrolled in 1986. Working on the discrete Nonlinear Schrödinger Equation

$$i\dot{Q} = Q_{n+1} - 2Q_n + Q_{n-1} + |Q_n|^{2\sigma}(Q_{n+1} + Q_{n-1}).$$

In particular, using a geometrical analysis and a Painlevé analysis to examine the behavior near infinity. Advisors: N. Ercolani and D. W. McLaughlin.

W. MACEVOY - B.S. 1989, Mesa State College, Grand Junction Colorado. Ph.D. student in Applied Mathematics. Working to study the dynamics of filament interaction in solutions to saturable NLS with two transverse dimensions, and in the optical ring cavity problem with two transverse dimensions. Advisor: A. C. Newell.

R. NORTHCUTT - B.S. 1985, Stanford University, Palo Alto, California. Ph.D. student in Applied Mathematics. Enrolled in 1987. Worked with Mac Hyman of LANL developing software for integration and analysis of systems of partial differential equations in arbitrary spatial dimensions. The package is currently in use both at the University of Arizona and at LANL. Currently working to continue the optical ring cavity calculations in two transverse dimensions that was started by Adachihara, Moloney and Newell. Advisor: David Levermore.

M. POKORNY - B.S. 1983, University of Arizona. Ph.D. student in Applied Mathematics. Enrolled in 1987. Working on computer simulations of percolation models. One model is related to invasion percolation on Z2, and another deals with the number of infinite clusters in percolation on a "stack" of Bethe lattices. Advisor: C. M. Newman.

A. OUARZEDDINI - B.S. 1982 and M.S 1983 Clarkson University, Potsdam, NY. Ph.D. student in Applied Mathematics. Enrolled in 1984. Worked with J.V. Moloney and Hatsuo Adachihara on the modulational instabilities in a nonlinear ring cavity. Performed numerical calculations in stimulated Raman Scattering to look at both self-focussing and Raman gain effects in the laser and the Stokes fields. Advisors: A. C. Newell and J. V. Moloney.

J. POWELL - B.S. 1985, Colorado State University. Ph.D. student in Applied Mathematics. Will graduate in 1990. Enrolled in 1985. Currently studying the propagation of fronts in Complex Ginzburg-Landau (CGL) type equations with quintic polynomial nonlinearities. Presented an argument for understanding asymptotic front speeds using asymptotic spatial dependence and found integrable fronts using the WTC (Weiss, Tabor and Carnevale) method to demonstrate that these fronts are predominant in physical parameter regimes. Presented a Liapunov functional argument for front stability in particular circumstances, extending these arguments to more general versions of the CGL and to higher dimensions. Advisor: A. C. Newell.

H. ROITNER - Diplomprüfung, 1987, University of Technology, Vienna, Austria. Ph.D. student in Applied Mathematics. Will graduate in 1990. Enrolled in 1987. Involved with a Kuramoto-Shivashinsky type perturbation of the Korteweg-deVries equation under periodic boundary conditions with methods from the general theory for nonlinear, dissipative PDE's (theory of inertial manifolds) as well as with the tools of the theory of completely integrable evolution equations (spectral transform), because the perturbation is assumed small (near-integrable case). Numerical studies of the equation for different perturbation parameters and dynamics applicable to perturbations of the Sine-Gordon and nonlinear Schrödinger equations. Advisors: N. Ercolani and D. W. McLaughlin.

C. SCHOBER - B.S. 1978 and M.S. 1985, Courant Institute, New York University, New York. Ph.D. student in Applied Mathematics. Enrolled in 1987. Working on the spectral transform for a discrete nonlinear Schrödinger equation, emphasizing homoclinic manifolds for this finite dimensional system. Advisor: D. W. McLaughlin.

J. TSAY - B.S. 1981, National Taiwan University, Taiwan. Ph.D. student in Applied Mathematics. Enrolled in 1987. Scattering theory for the discrete Schrödinger equation with a random potential having large support. Progress has been made for fixed energy with attempt to extend the result to wave packet with compact interval of energy. Advisor: W. B. Faris.

P. VARATHARAJAH - B.S. 1981, University of Jaffna, Sri Lanka; M.S. 1986, University of Arizona. Ph.D. student in Applied Mathematics. Enrolled in 1986. Continuing to work problems relating to optical beam propagation at the interface of two or more nonlinear materials. We have applied the equivalent particle theory to several problems where diffusive effects are considered. Currently primarily involved in extending the equivalent particle theory to the case of a 2-soliton solution propagating near the interface of two nonlinear material. Advisor: A. C. Newell.

C. WU - B.S. 1982, Guizhou University, People's Republic of China; M.S. 1984, Institute of Mathematical Sciences, Academia Sinica, People's Republic of China. Ph.D. student in Applied Mathematics. Enrolled in 1986. Worked on uniqueness of the infinite cluster in a half space and for directed percolation, the continuity of the connectivity functions and the percolation density for the random cluster model. Advisor. C. M. Newman.

L. ZHENG - B.S. 1982, Sichuan University, People's Republic of China; M.S. 1984, Institute of Math Sciences, People's Republic of China. Ph.D. student in Mathematics. Enrolled in 1986. Started to investigate the relation between entropy and Lyapunov exponents for noninvertible mappings. Advisor: L. Young.

M. ZOU - B.S. and M.S., University of Science & Technology, Poeple's Republic of China. Ph.D. student in Mathematics. Enrolled in 1986. Found examples clarifying the "weak Painlevé property" and its connections (via Ziglin's theory) to the existence of additional constants of motion of Hamiltonian systems. Advisor: H. Flaschka.

VII. LIST OF ACMS REPRINTS AND PREPRINTS

ACMS Publication 86-1
    J-G. Caputo, W. Faris, A. C. Newell, C. M. Newman, M. Shelley
    Nonlinear Tunneling Through Random Media.
    To be submitted.
ACMS Publication 86-2
    A. C. Newell
    Chaos and Turbulence. Proc. Wood's Hole Summer Seminar in
    Geographical Fluid Dynamics on "Shear Flow Turbulence,"
    WHOI-86-45, 90-103 (1986).
ACMS Publication 86-3
    A. C. Newell
    Chaos and Turbulence: Is There a Connection? Special Proc. of Conf. on Mathematics Applied to
    Fluid Mechanics and Stability Dedicated in Memory of Richard C. DiPrima.
    SIAM 157-189, (1986).
ACMS Publication 86-4
    A. C. Newell, M. Tabor, Y. B. Zeng
    A Unified Approach to Painlevé Expansions.
    Phys. 29D, 1-68 (1987).
ACMS Publication 86-5
    J. D. Gibbon, A. C. Newell, M. Tabor, Y. B. Zeng
    Lax Pairs, Bäcklund Transformations and Special Solutions for Ordinary Differential Equations.
    Nonlinearity 1, 1-10 (1988).
AMCS Publication 86-6
    A. C. Newell, Z. Yunbo
    The Hirota Conditions.
    J. Math. Phys 27, 2016-2021 (1986).
ACMS Publication 86-7
    L. Chierchia, N. Ercolani, D. W. McLaughlin
    On the Weak Limit of Rapidly Ocillating Waves.
    Duke Math J. 55, 4 (1987).
ACMS Publication 86-8
    N. Ercolani, M. G. Forest, D. W. McLaughlin, R. Montgomery
    Hamiltonian Structure for the Modulation Equations of a Sine-Gordon Wavetrain.
    Duke Math J. 55, 4 (1988).
ACMS Publication 86-9
    A. Mazor, A. R. Bishop, D. W. McLaughlin
    *Phase-Pulling and Space-Time Complexity in an AC Driven Damped One-
    Dimensional Sine-Gordon System.*
    Phys. Lett. A 119 (6), 273-279 (1987).
ACMS Publication 86-10
    A. Bishop, D. W. McLaughlin, E. A. Overman II
    *A Quasi-Periodic Route to Chaos in a Near-Integrable Partial Differential
    Equation: Homoclinic Crossings.*
    Phys. Lett. A 127, 335-340 (1988).
ACMS Publication 86-11
    A. R. Bishop, M. G. Forest, D. W. McLaughlin, E. A. Overman II
    A Quasi-Periodic Route to Chaos in a Near-Integrable PDE.
    Physica D, 293-328 (1986).

ACMS Publication 86-12
  N. Ercolani, D. W. McLaughlin, M. G. Forest
  Homoclinic Orbits for the Periodic Sine-Gordon Equation.
  Accepted for *Physica D (1988)*.
ACMS Publication 86-13
  N. Ercolani, D. W. McLaughlin, M. G. Forest
  Geometry of the Modulational Instability: Part I. Local Results, Part II.
  Global Results.
  To appear in *Memoirs of the AMS*.
ACMS Publication 86-14
  H. Adachihara, D. W. McLaughlin, J. V. Moloney, A. C. Newell
  Solitary Waves as Fixed Points of Infinite-Dimensional Maps for an Optical
  Biostable Ring Cavity: Analysis.
  J. Math. Phys. 29(1), 63-85 (1988).
ACMS Publication 86-15
  A. Aceves, H. Adachihara, C. Jones, J. C. Lerman, D. W. Mclaughlin, J. V. Moloney,
  A. C. Newell.
  Chaos and Coherent Structure in Partial Differential Equations.
  Physica 18D, 85-112 (1986).
ACMS Publication 86-16
  A. Aceves, J. V. Moloney, A. C. Newell
  Reflection, Transmission and Stability Characteristics of Optical Beams
  Incident at a Nonlinear Dielectric Interfaces.
  J. of the Optical Society of America B. Special issue on Nonlinear Waveguides 5, 559 (1988).
ACMS Publication 86-17
  C. M. Newman
  Another Critical Exponent Inequality for Percolation: $\beta \geq s/\delta$.
  J. Stat. Phys. 47, 695-699 (1987).
ACMS Publication 86-18
  H. R. Brand, P. S. Lomdahl, A. C. Newell
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